Problem Solutions

By
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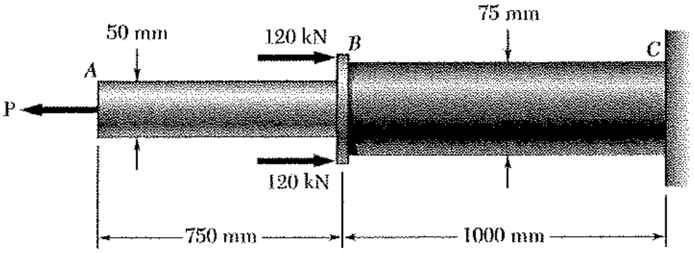
Chapter 1

Problem 1.1

1.1 Two solid cylindrical rods *AB* and *BC* are welded together at B and loaded as shown. Determine the magnitude of the force P for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.



$$A_{AB} = \frac{\pi}{4}(50)^2 = 1963.5\ mm^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{1963.5}$$

$$= 509.3 \times 10^{-6} P$$

$$A_{BC} = \frac{\pi}{4}(75)^2 = 4417.9\ mm^2$$

$$\sigma_{BC} = \frac{(2)(120) - P}{A_{AB}}$$

$$= \frac{240 - P}{4417.9} = 0.0543 - 226.4 \times 10^{-6} P$$

Equating σ_{AB} to $2\,\sigma_{BC}$

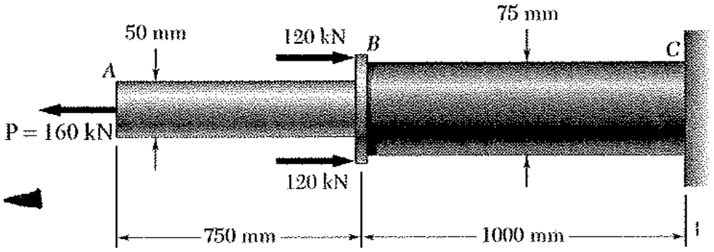
$$509.3 \times 10^{-6} P = 2(0.0543 - 226.4 \times 10^{-6} P)$$

$P = 112.9\ kN$ ◄

Problem 1.2

1.2 In Prob. 1.1, knowing that $P = 160$ kN, determine the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

1.1 Two solid cylindrical rods *AB* and *BC* are welded together at B and loaded as shown. Determine the magnitude of the force P for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.



(a) Rod AB.

$P = 160\ kN$ (tension)

$$A_{AB} = \frac{\pi d_{AB}^2}{4} = \frac{\pi (50)^2}{4} = 1963.5\ mm^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{160 \times 10^3}{1963.5} = 81.5\ MPa$$ ◄

(b) Rod BC.

$F = 160 - (2)(120) = -80\ kN$ i.e. 80 kN compression.

$$A_{BC} = \frac{\pi d_{BC}^2}{4} = \frac{\pi (75)^2}{4} = 4417.9\ mm^2$$

$$\sigma_{BC} = \frac{F}{A_{BC}} = \frac{-80 \times 10^3}{4417.9}$$

$\sigma_{BC} = -18.1\ MPa$ ◄

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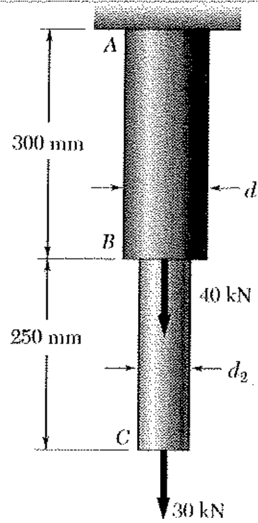
Problem 1.3

1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the average normal stress must not exceed 175 MPa in rod *AB* and 150 MPa in rod *BC*, determine the smallest allowable values of d_1 and d_2.



Rod AB.

$$P = 40 + 30 = 70 \text{ kN} = 70\times10^3 \text{ N}$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{\frac{\pi}{4}d_1^2} = \frac{4P}{\pi d_1^2}$$

$$d_1 = \sqrt{\frac{4P}{\pi\sigma_{AB}}} = \sqrt{\frac{(4)(70\times10^3)}{\pi(175\times10^6)}} = 22.6\times10^{-3} \text{ m}$$

$$d_1 = 22.6 \text{ mm}$$

Rod BC.

$$P = 30 \text{ kN} = 30\times10^3 \text{ N}$$

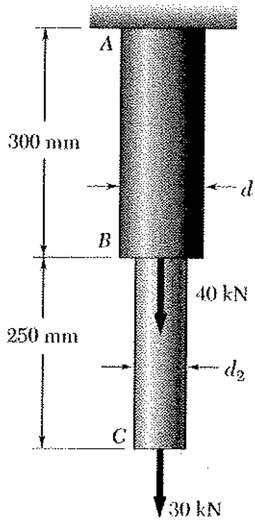
$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{P}{\frac{\pi}{4}d_2^2} = \frac{4P}{\pi d_2^2}$$

$$d_2 = \sqrt{\frac{4P}{\pi\sigma_{BC}}} = \sqrt{\frac{(4)(30\times10^3)}{\pi(150\times10^6)}} = 15.96\times10^{-3} \text{ m}$$

$$d_2 = 15.96 \text{ mm}$$

Problem 1.4

1.4 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d_1 = 50$ mm and $d_2 = 30$ mm, find average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.



(a) Rod AB.

$$P = 40 + 30 = 70 \text{ kN} = 70\times10^3 \text{ N}$$

$$A = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(50)^2 = 1.9635\times10^3 \text{ mm}^2 = 1.9635\times10^{-3} \text{ m}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{70\times10^3}{1.9635\times10^{-3}} = 35.7\times10^6 \text{ Pa}$$

$$\sigma_{AB} = 35.7 \text{ MPa}$$

(b) Rod BC.

$$P = 30 \text{ kN} = 30\times10^3 \text{ N}$$

$$A = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86\times10^{-6} \text{ m}^2$$

$$\sigma_{BC} = \frac{P}{A} = \frac{30\times10^3}{706.86\times10^{-6}} = 42.4\times10^6 \text{ Pa}$$

$$\sigma_{BC} = 42.4 \text{ MPa}$$

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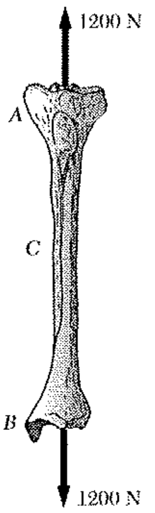
Problem 1.5

1.5 A strain gage located at *C* on the surface of bone *AB* indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at *C* to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at *C*.



$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $A = \frac{\pi}{4}(d_1^2 - d_2^2)$

$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

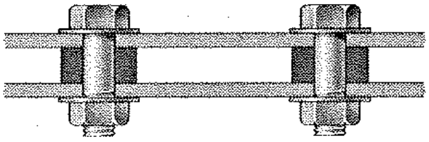
$$d_2^2 = (25\times10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80\times10^6)}$$

$$= 222.9\times10^{-6} \text{ m}^2$$

$$d_2 = 14.93\times10^{-3} \text{ m} \qquad d_2 = 14.93 \text{ mm}$$

Problem 1.6

1.6 Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.



At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time the spacer pushes that plate upward with a compressive force P_s. In order to maintain equilibrium

$$P_b = P_s$$

For the bolt, $\sigma_b = \frac{F_b}{A_b} = \frac{4P_b}{\pi d_b^2}$ or $P_b = \frac{\pi}{4}\sigma_b d_b^2$

For the spacer, $\sigma_s = \frac{P_s}{A_s} = \frac{4P_s}{\pi(d_s^2 - d_b^2)}$ or $P_s = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

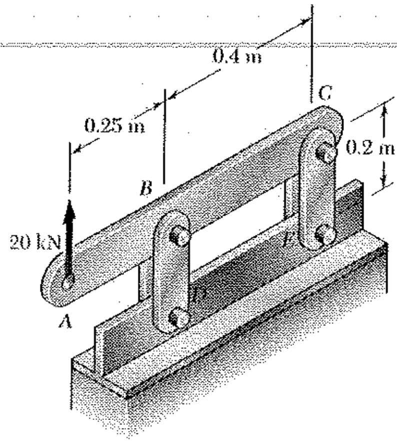
Equating P_b and P_s,

$$\frac{\pi}{4}\sigma_b d_b^2 = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$$

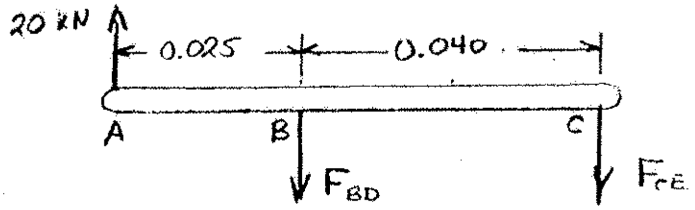
$$d_s = \sqrt{\left(1 + \frac{\sigma_b}{\sigma_s}\right)}\, d_b = \sqrt{1 + \frac{200}{130}}\,(16) \qquad d_s = 25.2 \text{ mm}$$

Problem 1.7

1.7 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.



Use bar ABC as a free body.



$\Sigma M_C = 0$: $(0.040)F_{BD} - (0.025 + 0.040)(20\times10^3) = 0$

$F_{BD} = 32.5\times10^3$ N   Link BD is in tension.

$\Sigma M_B = 0$: $-(0.040)F_{CE} - (0.025)(20\times10^3) = 0$

$F_{CE} = -12.5\times10^3$ N   Link CE is in compression.

Net area of one link for tension = $(0.008)(0.036 - 0.016)$

$= 160\times10^{-6}\ \text{m}^2$. For two parallel links, $A_{net} = 320\times10^{-6}\ \text{m}^2$

Tensile stress in link BD.

(a) $\sigma_{BD} = \dfrac{F_{BD}}{A_{net}} = \dfrac{32.5\times10^3}{320\times10^{-6}} = 101.56\times10^6$ $\sigma_{BD} = 101.6$ MPa ◄

Area for one link in compression = $(0.008)(0.036)$

$= 288\times10^{-6}\ \text{m}^2$. For two parallel links, $A = 576\times10^{-6}\ \text{m}^2$

(b) $\sigma_{CE} = \dfrac{F_{CE}}{A} = \dfrac{-12.5\times10^3}{576\times10^{-6}} = -21.70\times10^6$ $\sigma_{CE} = -21.7$ MPa ◄

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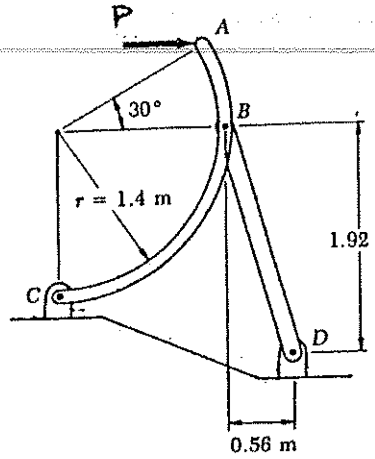
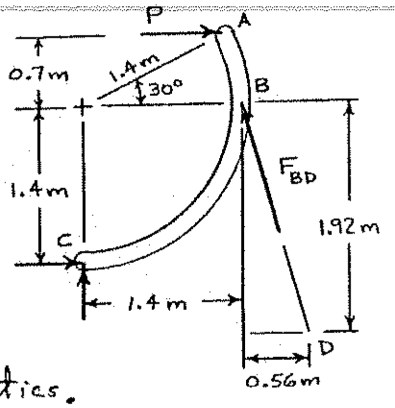
Problem 1.8

1.8 Knowing that the central portion of the link *BD* has a uniform cross-sectional area of 800 mm², determine the magnitude of the load **P** for which the normal stress in that portion of *BD* is 50 MPa.





$$F_{BD} = \sigma A$$
$$= (50\times10^{6})(800\times10^{-6})$$
$$= 40\times10^{3}\ N$$

$$BD = \sqrt{(0.56)^2 + (1.92)^2}$$
$$= 2.00\ m$$

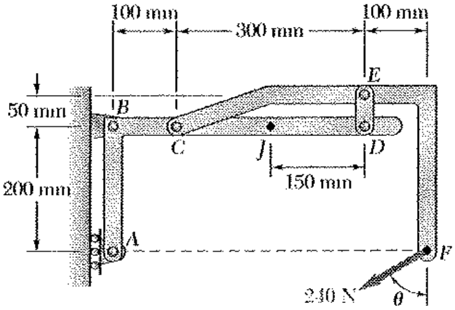
Use Free Body AC for statics.

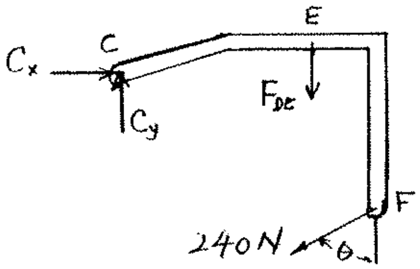
$+\circlearrowleft \Sigma M_C = 0:$ $$\frac{0.56}{2.00}(40\times10^3)(1.4) + \frac{1.92}{2.00}(40\times10^3)(1.4) - P(0.7+1.4) = 0$$

$P = 33.1\times10^3\ N$ $\qquad$ $P = 33.1\ kN$ ◄

Problem 1.9

1.9 Knowing that link *DE* is 25 mm wide and 3 mm thick, determine the normal stress in the central portion of that link when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.





Use member CEF as a free body

$+\circlearrowleft \Sigma M_C = 0$

$$-0.3\,F_{DE} - (0.2)(240\sin\theta) - (0.4)(240\cos\theta) = 0$$

$$F_{DE} = -160\sin\theta - 320\cos\theta\ N$$

$$A_{DE} = (0.025)(0.003) = 75\times10^{-6}\ m^2$$

$$\sigma_{DE} = \frac{F_{DE}}{A_{DE}}$$

(a) $\theta = 0$: $\quad F_{DE} = -320\ N$

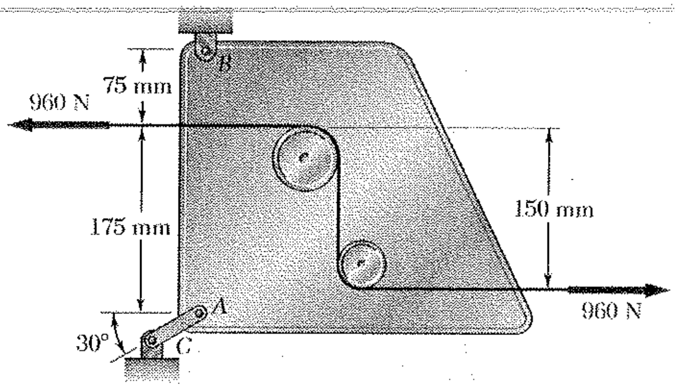
$$\sigma_{DE} = \frac{-320}{75\times10^{-6}} = -4.27\ MPa$$ ◄

(b) $\theta = 90°$: $\quad F_{DE} = -160\ N$

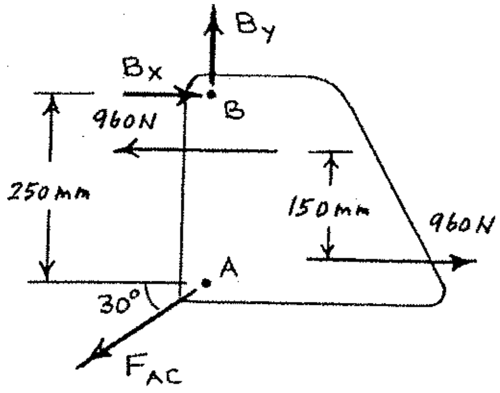
$$\sigma_{DE} = \frac{-160}{75\times10^{-6}} = -2.13\ MPa$$ ◄

Problem 1.10

1.10 Link *AC* has a uniform rectangular cross section 3 mm thick and 12 mm wide. Determine the normal stress in the central portion of the link.



Free Body Diagram of Plate



Note that the two 960-N forces form a couple of moment

$(960\ N)(0.15\ m) = 144\ N.m$

$\circlearrowleft \Sigma M_B = 0: \quad 144\ N.m - (F_{AC}\cos 30°)(0.25\ m) = 0$

$F_{AC} = 665.1\ N$

Area of link: $A_{AC} = (3\ mm)(12\ mm) = 36\ mm^2$

Stress $\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{665.1}{36} = 18.475\ MPa \qquad \sigma_{AC} = 18.5\ MPa$ ◀

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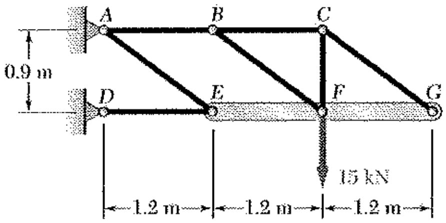
Problem 1.11

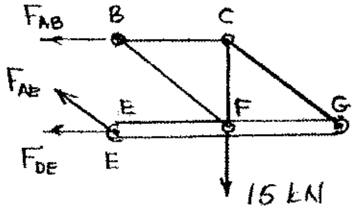
1.11 The rigid bar *EFG* is supported by the truss system shown. Knowing that the member *CG* is a solid circular rod of 18 mm diameter, determine the normal stress in *CG*.



Using portion EFGCB as a free body

$+\uparrow \Sigma F_y = 0: \frac{0.9}{1.5} F_{AB} - 15 = 0$

$F_{AE} = 25 \text{ kN}$



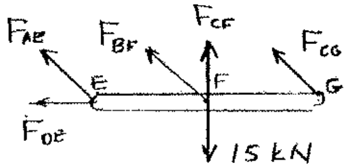
Using beam EFG as a free body

$\circlearrowleft M_F = 0: -(1.2)\frac{0.9}{1.2} F_{AE} + (1.2)\left(\frac{0.9}{1.2} F_{CG}\right) = 0$

$F_{CG} = F_{AE} = 25 \text{ kN}$

Cross sectional area of member CG

$A_{CG} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.018)^2 = 254.4 \times 10^{-6} \text{ m}^2$

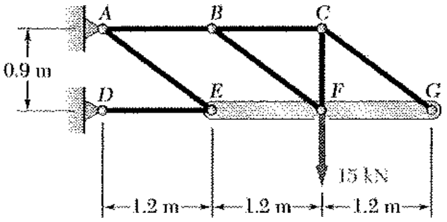


Normal stress in CG.

$\sigma_{CG} = \frac{F_{CG}}{A_{CG}} = \frac{25}{254.4 \times 10^{-6}} = 98.3 \text{ MPa}$ ◄

Problem 1.12

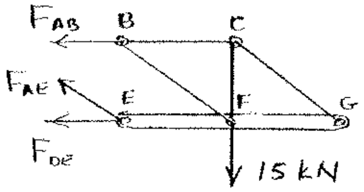
1.12 The rigid bar *EFG* is supported by the truss system shown. Determine the cross-sectional area of member *AE* for which the normal stress in the member is 105 MPa.



Using portion EFGCB as a free body

$+\uparrow \Sigma F_y = 0: \frac{0.9}{1.5} F_{AE} - 15 = 0$

$F_{AE} = 25 \text{ kN}$



Stress in member AE $\qquad \sigma_{AE} = 105 \text{ MPa}$

$\sigma_{AE} = \frac{F_{AE}}{A_{AE}}$

$A_{AE} = \frac{F_{AC}}{\sigma_{AE}} = \frac{25 \times 10^3}{105 \times 10^{-6}} = 238.1 \times 10^{-6} \text{ m}^2$ ◄

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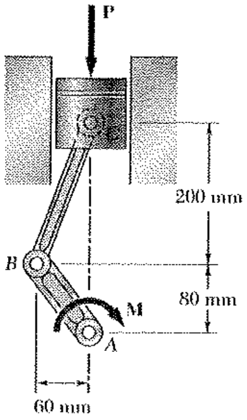
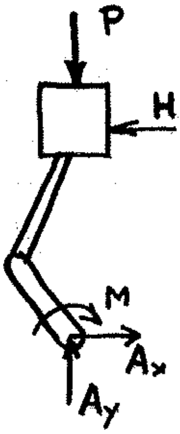
Problem 1.13

1.13 A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm^2 uniform cross section.



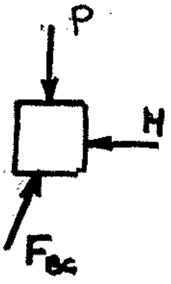
Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y.

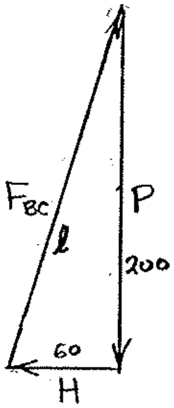


$\circlearrowleft \Sigma M_A = 0:$

$(0.280\text{ m})H - 1500\text{ N·m} = 0$

$H = 5.3571 \times 10^3\text{ N}$





Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{BC} is known. Draw the force triangle and solve for P and F_{BC} by proportions.

$l = \sqrt{200^2 + 60^2} = 208.81\text{ mm}$

$\dfrac{P}{H} = \dfrac{200}{60} \quad \therefore \quad P = 17.86 \times 10^3\text{ N}$

(a) $P = 17.86\text{ kN}$ ◄

$\dfrac{F_{BC}}{H} = \dfrac{208.81}{60} \quad \therefore \quad F_{BC} = 18.643 \times 10^3\text{ N}$

Rod BC is a compression member. Its area is $450\text{ mm}^2 = 450 \times 10^{-6}\text{ m}^2$

Stress, $\sigma_{BC} = \dfrac{-F_{BC}}{A} = \dfrac{-18.643 \times 10^3}{450 \times 10^{-6}} = -41.4 \times 10^6\text{ Pa}$

(b) $\sigma_{BC} = -41.4\text{ MPa}$ ◄

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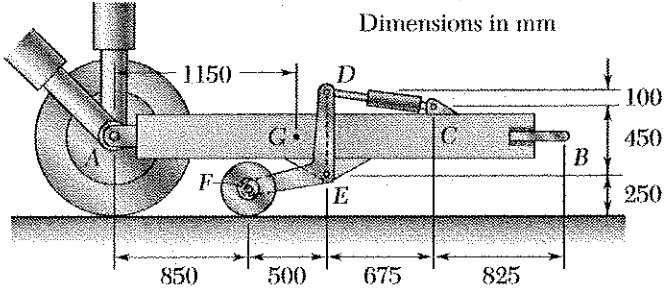
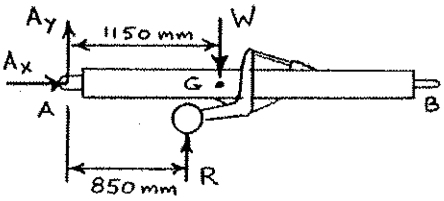
Problem 1.14

1.14 An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units *DEF*. The mass of the entire tow bar is 200 kg, and its center of gravity is located at *G*. For the position shown, determine the normal stress in the rod.



FREE BODY – ENTIRE TOW BAR:



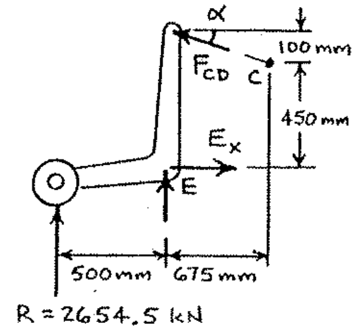
$W = (200\ \text{kg})(9.81\ \text{m/s}^2) = 1962.00\ \text{N}$

$+\circlearrowleft \Sigma M_A = 0:$

$850R - 1150(1962.00\ \text{N}) = 0$

$R = 2654.5\ \text{N}$

FREE BODY – BOTH ARM & WHEEL UNITS:



$\tan\alpha = \frac{100}{675} \qquad \alpha = 8.4270°$

$+\circlearrowleft \Sigma M_E = 0:$

$(F_{CD}\cos\alpha)(550) - R(500) = 0$

$F_{CD} = \frac{500}{550\cos 8.4270°}(2654.5\ \text{N})$

$= 2439.5\ \text{N} \quad (\text{COMP.})$

$\sigma_{CD} = -\frac{F_{CD}}{A_{CD}} = -\frac{2439.5\ \text{N}}{\pi(0.0125\ \text{m})^2}$

$= -4.9697 \times 10^6\ \text{Pa}$

$\sigma_{CD} = -4.97\ \text{MPa}$ ◀

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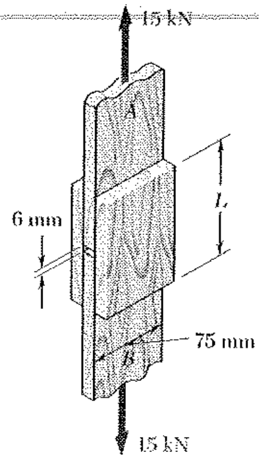
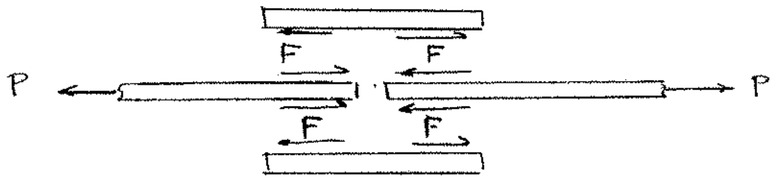
Problem 1.15

1.15 The wooden members *A* and *B* are to be joined by plywood splice plates that will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be 6 mm, determine the smallest allowable length *L* if the average shearing stress in the glue is not to exceed 700 kPa.



There are four separate areas that are glued. Each of these areas transmits one half the the 15 kN load. Thus

$$F = \tfrac{1}{2}P = \tfrac{1}{2}(15) = 7.5 \text{ kN} = 7500 \text{ N}$$



Let l = length of one glued area and $w = 75 \text{ mm} = 0.075 \text{ m}$ be its width.

For each glued area, $A = lw$

Average shearing stress: $\tau = \dfrac{F}{A} = \dfrac{F}{lw}$

The allowable shearing stress is $\tau = 700 \times 10^3$ Pa

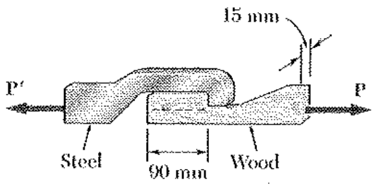
Solving for l, $l = \dfrac{F}{\tau w} = \dfrac{7500}{(700\times10^3)(0.075)} = 0.14286 \text{ m} = 142.85 \text{ mm}$

Total length L: $L = l + (\text{gap}) + l = 142.85 + 6 + 142.85$

$L = 292 \text{ mm}$ ◀

Problem 1.16

1.16 When the force **P** reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.



Area being sheared

$A = 90 \text{ mm} \times 15 \text{ mm} = 1350 \text{ mm}^2 = 1350 \times 10^{-6} \text{ m}^2$

Force $P = 8 \times 10^3$ N

Shearing stress $\tau = \dfrac{P}{A} = \dfrac{8\times10^3}{1350\times10^{-6}} = 5.93 \times 10^3 \text{ Pa} = 5.93 \text{ MPa}$ ◀

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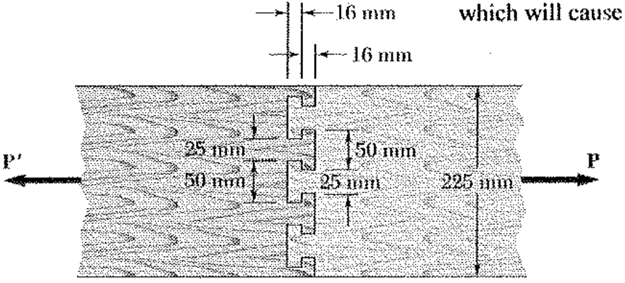
Problem 1.17

1.17 Two wooden planks, each 12 mm thick and 225 mm wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 8 MPa, determine the magnitude *P* of the axial load which will cause the joint to fail.



Six areas must be sheared off when the joint fails. Each of these areas has dimensions 16 mm × 12 mm, its area being

$$A = (16)(12) = 192\ \text{mm}^2 = 192\times10^{-6}\ \text{m}^2$$

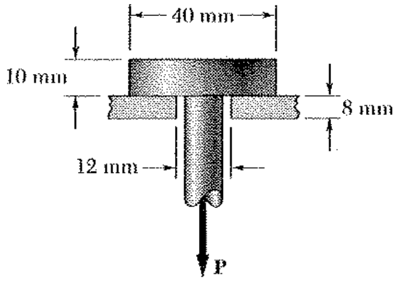
At failure the force F carried by each of areas is

$$F = \tau A = (8\times10^6)(192\times10^{-6}) = 1536\ \text{N} = 1.536\ \text{kN}$$

Since there are six failure areas $\quad P = 6F = (6)(1.536) = 9.22\ \text{kN}$

Problem 1.18

1.18 A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load **P** that can be applied to the rod.



For the steel rod,

$$A_1 = \pi d_1 t_1 = (\pi)(0.012)(0.010)$$
$$= 376.99\times10^{-6}\ \text{m}^2$$

$$\tau_1 = \frac{P}{A_1} \rightarrow P_1 = \tau_1 A_1$$

$$P_1 = (180\times10^6)(376.99\times10^{-6}) = 67.86\times10^3\ \text{N}$$

For the aluminum plate,

$$A_2 = \pi d_2 t_2 = (\pi)(0.040)(0.008) = 1.00531\times10^{-3}\ \text{m}^2$$

$$\tau_2 = \frac{P_2}{A_2} \Rightarrow P_2 = \tau_2 A_2$$

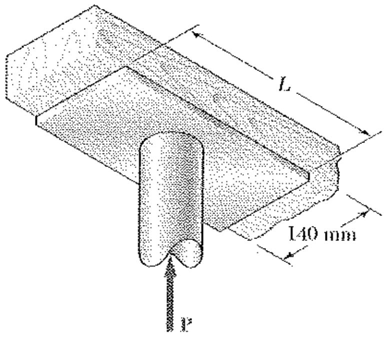
$$P_2 = (70\times10^6)(1.0053\times10^{-6}) = 70.372\times10^3\ \text{N}$$

The limiting value for the load P is the smaller of P_1 and P_2.

$P = 67.86\times10^3$ N $\qquad\qquad P = 67.9$ kN

Problem 1.19

1.19 The axial force in the column supporting the timber beam shown is $P = 75$ kN. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 3.0 MPa.



SOLUTION

$$\sigma_b = \frac{P}{A} = \frac{P}{Lw}$$

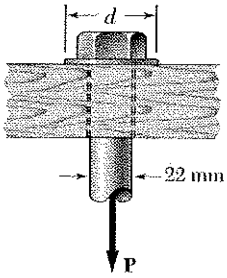
Solving for L: $L = \frac{P}{\sigma_b w} = \frac{75\times10^3}{(3.0\times10^6)(0.140)}$

$$178.6\times10^{-3}\ \text{m}$$

$$L = 178.6\ \text{mm}$$ ◀

Problem 1.20

1.20 The load **P** applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter d of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa.



Steel rod: $A = \frac{\pi}{4}(0.022)^2 = 380.13\times10^{-6}\ \text{m}^2$

$\sigma = 35\times10^6$ Pa

$P = \sigma A = (35\times10^6)(380.13\times10^{-6})$

$= 13.305\times10^3$ N

Washer: $\sigma_b = 5\times10^6$ Pa

Required bearing area:

$$A_b = \frac{P}{\sigma_b} = \frac{13.305\times10^3}{5\times10^6} = 2.6609\times10^{-3}\ \text{m}^2$$

But, $A_b = \frac{\pi}{4}(d^2 - d_i^2)$

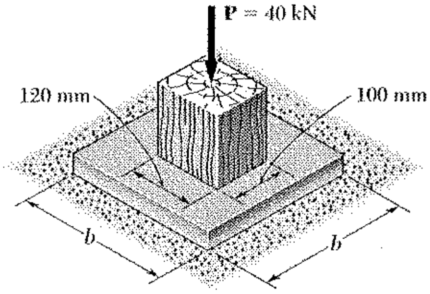
$$d^2 = d_i^2 + \frac{4A_b}{\pi}$$

$$= (0.025)^2 + \frac{(4)(2.6609\times10^{-3})}{\pi}$$

$$= 4.013\times10^{-3}\ \text{m}^2$$

$d = 63.3\times10^{-3}$ m $\qquad d = 63.3$ mm ◀

Problem 1.21



1.21 A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

(a) Bearing stress on concrete footing.

$P = 40\ \text{kN} = 40\times10^3\ \text{N}$

$A = (100)(120) = 12\times10^3\ \text{mm}^2 = 12\times10^{-3}\ \text{m}^2$

$\sigma = \dfrac{P}{A} = \dfrac{40\times10^3}{12\times10^{-3}} = 3.33\times10^6\ \text{Pa}$

3.33 MPa ◄

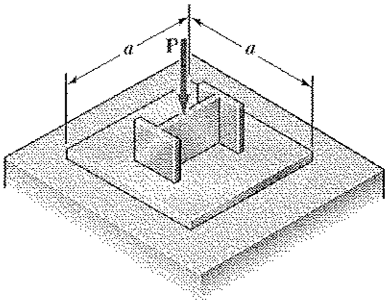
(b) Footing area. $P = 40\times10^3\ \text{N}$ $\sigma = 145\ \text{kPa} = 45\times10^3\ \text{Pa}$

$\sigma = \dfrac{P}{A}$ $A = \dfrac{P}{\sigma} = \dfrac{40\times10^3}{145\times10^3} = 0.27586\ \text{m}^2$

Since the area is square, $A = b^2$

$b = \sqrt{A} = \sqrt{0.27586} = 0.525\ \text{m}$ $b = 525\ \text{mm}$ ◄

Problem 1.22



1.22 An axial load **P** is supported by a short W200 × 59 column of cross-sectional area A = 7560 mm² and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 200 MPa and that the bearing stress on the concrete foundation must not exceed 20 MPa, determine the side *a* of the plate that will provide the most economical and safe design.

For the column $\sigma = \dfrac{P}{A}$

or $P = \sigma A = (200\times10^6)(7560\times10^{-6}) = 1512\ kN$

For the $a\times a$ plate, $\sigma = 20\ MPa$

$A = \dfrac{P}{\sigma} = \dfrac{1.512}{20} = 0.0756/m^2$

Since the plate is square $A = a^2$

$a = \sqrt{A} = \sqrt{0.0756} = 0.275\ m$

$= 275\ mm$

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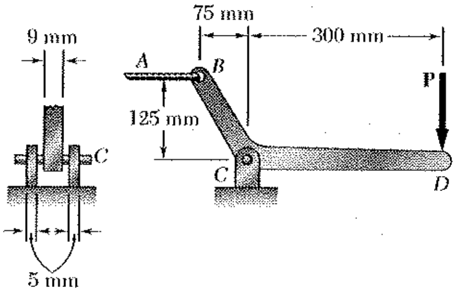
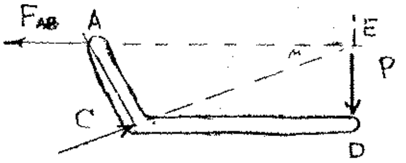
Problem 1.23

1.23 A 6-mm-diameter pin is used at connection C of the pedal shown. Knowing that $P = 500$ N, determine (*a*) the average shearing stress in the pin, (*b*) the nominal bearing stress in the pedal at C, (*c*) the nominal bearing stress in each support bracket at C.



Draw free body diagram of ACD. Since ACD is a 3-force member, the reaction at C is directed toward point E, the intersection of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325$ mm.

$+\uparrow \Sigma F_y = 0: \quad \frac{125}{325}C - P = 0 \qquad C = 2.6P = (2.6)(500) = 1300$ N

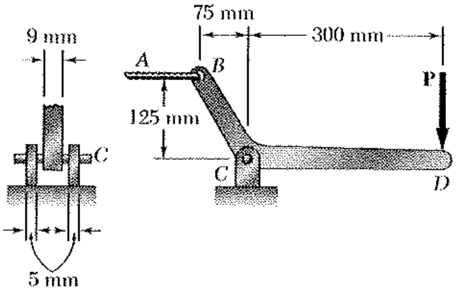
(a) $\tau_{pin} = \frac{\frac{1}{2}C}{A_{pin}} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} = \frac{2C}{\pi d^2} = \frac{(2)(1300)}{\pi(6\times10^{-3})^2} = 23.0\times10^6$ Pa $\qquad \tau_{pin} = 23.0$ MPa ◀

(b) $\sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{1300}{(6\times10^{-3})(9\times10^{-3})} = 24.1\times10^6$ Pa $\qquad \sigma_b = 24.1$ MPa ◀

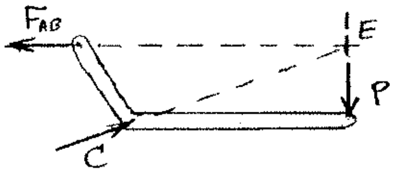
(c) $\sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{1300}{(2)(6\times10^{-3})(5\times10^{-3})} = 21.7\times10^6$ Pa $\qquad \sigma_b = 21.7$ MPa ◀

Problem 1.24

1.24 Knowing that a force **P** of magnitude 750 N is applied to the pedal shown, determine (*a*) the diameter of the pin at C for which the average shearing stress in the pin is 40 MPa, (*b*) the corresponding bearing stress in the pedal at C, (*c*) the corresponding bearing stress in the each support bracket at C.



Draw free body diagram of ACD. Since ACD is a 3-force member, the reaction at C is directed toward point E, the intersection of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325$ mm

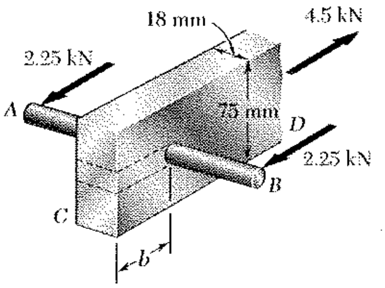
$+\uparrow \Sigma F_y = 0: \quad \frac{125}{325}C - P = 0 \qquad C = 2.6P = (2.6)(750) = 1950$ N

(a) $\tau_{pin} = \frac{\frac{1}{2}C}{A_{pin}} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} \qquad d = \sqrt{\frac{2C}{\pi\tau_{pin}}} = \sqrt{\frac{(2)(1950)}{\pi(40\times10^6)}} = 5.57\times10^{-3}$ m $\qquad d = 5.57$ mm ◀

(b) $\sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{1950}{(5.57\times10^{-3})(9\times10^{-3})} = 38.9\times10^6$ Pa $\qquad \sigma_b = 38.9$ MPa ◀

(c) $\sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{1950}{(2)(5.57\times10^{-3})(5\times10^{-3})} = 35.0\times10^6$ Pa $\qquad \sigma_b = 35.0$ MPa ◀

Problem 1.25



1.25 A 12-mm-diameter steel rod *AB* is fitted to a round hole near end *C* of the wooden member *CD*. For the loading shown, determine (*a*) the maximum average normal stress in the wood, (*b*) the distance *b* for which the average shearing stress is 620 kPa on the surfaces indicated by the dashed lines, (*c*) the average bearing stress on the wood.

(a) Maximum average normal stress in the wood.

$A_{net} = (75-12)(18) = 1.134\times10^{3}\ \text{mm}^2 = 1.134\times10^{-3}\ \text{m}^2$

$P = 4.50\ \text{kN} = 4.50\times10^{3}\ \text{N}$

$\sigma = \frac{P}{A_{net}} = \frac{4.50\times10^{3}}{1.134\times10^{-3}} = 3.97\times10^{6}\ \text{Pa}$ 3.97 MPa ◄

(b) $\tau = \frac{P}{A} = \frac{P}{2bt}$ $b = \frac{P}{2t\tau} = \frac{4.50\times10^{3}}{(2)(18\times10^{-3})(620\times10^{3})} = 202\times10^{-3}\ \text{m}$

$b = 202\ \text{mm}$ ◄

(c) $\sigma_b = \frac{P}{dt} = \frac{4.50\times10^{3}}{(12\times10^{-3})(18\times10^{-3})} = 20.8\times10^{6}\ \text{Pa}$ 20.8 MPa ◄

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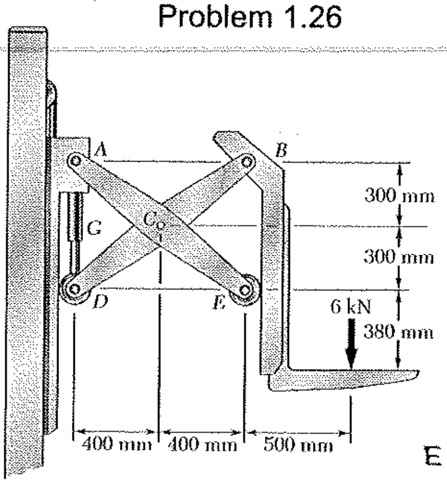
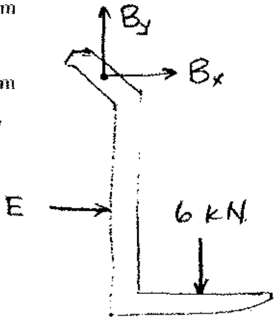
Problem 1.26



1.26 Two identical linkage-and-hydraulic-cylinder systems control the position of the forks of a fork-lift truck. The load supported by the one system shown is 6 kN. Knowing that the thickness of member *BD* is 16 mm, determine (*a*) the average shearing stress in the 12-mm-diameter pin at *B*, (*b*) the bearing stress at *B* in member *BD*.



Use one fork as a free body.

$+\circlearrowright \sum M_B = 0:$

$0.6E - (0.5)(6) = 0$

$E = 5$ kN →

$\overset{+}{\rightarrow} \sum F_x = 0$

$E + B_x = 0 \qquad B_x = -E$

$B_x = 5$ kN ←

$+\uparrow \sum F_y = 0: \quad B_y - 6 = 0 \qquad B_y = 6$ kN

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{5^2 + 6^2} = 7.81$ kN

(a) Shearing stress in pin at B.

$A_{pin} = \frac{\pi}{4} d_{pin}^2 = \frac{\pi}{4}(0.012)^2 = 113.1 \times 10^{-6} \ m^2$

$\tau = \frac{B}{A_{pin}} = \frac{7.81 \times 10^3}{113.1 \times 10^{-6}} = 69 \ MPa$

(b) Bearing stress at B.

$\sigma = \frac{B}{dt} = \frac{7.8 \times 10^3}{(0.012)(0.016)} = 40.6 \ MPa$

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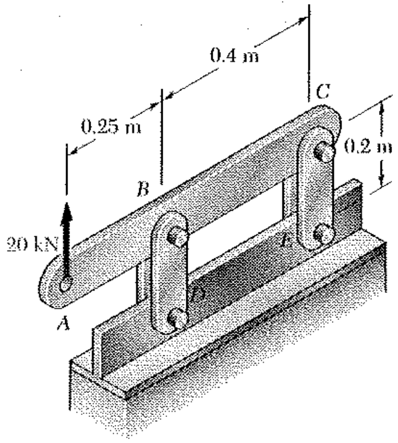
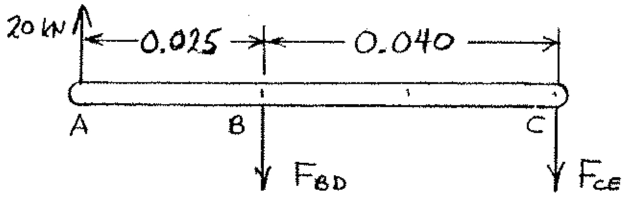
Problem 1.27



1.27 For the assembly and loading of Prob. 1.7, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in member *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10 × 50-mm uniform rectangular cross section.

1.7 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

Use bar ABC as a free body.



$$\circlearrowleft \Sigma M_C = 0: \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N}$$

(a) Shear pin at B. $\tau = \dfrac{F_{BD}}{2A}$ for double shear,

where $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \qquad \tau = 80.8 \text{ MPa} \blacktriangleleft$$

(b) Bearing: link BD. $A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{ m}^2$

$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \qquad \sigma_b = 127.0 \text{ MPa} \blacktriangleleft$$

(c) Bearing in ABC at B.

$$A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \qquad \sigma_b = 203 \text{ MPa} \blacktriangleleft$$

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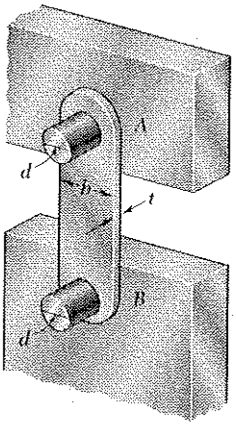
Problem1.28

1.28 Link *AB*, of width $b = 50$ mm and thickness $t = 6$ mm, is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is −140 MPa, and that the average shearing stress in each of the two pins is 80 MPa, determine (*a*) the diameter *d* of the pins, (*b*) the average bearing stress in the link.



Rod AB is in compression.

$A = bt$ where $b = 50\,\text{mm}$ and $t = 6\,\text{mm}$

$A = (0.050)(0.006) = 300 \times 10^{-6}\ \text{m}^2$

$P = -\sigma A = -(-140 \times 10^6)(300 \times 10^{-6})$

$= 42 \times 10^3\ \text{N}$

For the pin, $A_p = \frac{\pi}{4} d^2$ and $\tau = \dfrac{P}{A_p}$

$A_p = \dfrac{P}{\tau} = \dfrac{42 \times 10^3}{80 \times 10^6} = 525 \times 10^{-6}\ \text{m}^2$

(a) Diameter d.

$d = \sqrt{\dfrac{4A_p}{\pi}} = \sqrt{\dfrac{(4)(525 \times 10^{-6})}{\pi}} = 2.585 \times 10^{-3}\ \text{m}$

$d = 25.9\ \text{mm}$ ◄

(b) $\sigma_b = \dfrac{P}{dt} = \dfrac{42 \times 10^3}{(25.85 \times 10^{-3})(0.006)} = 271 \times 10^6\ \text{Pa}$

$\sigma_b = 271\ \text{MPa}$ ◄

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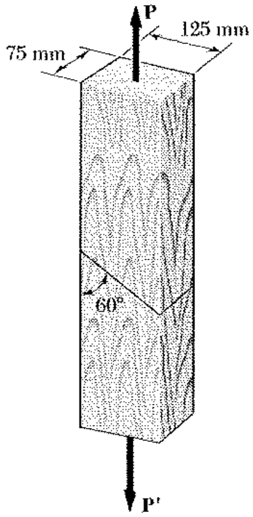
Problem 1.29



1.29 The 5.6-kN load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

$P = 6227 N$ $\qquad \theta = 90° - 60° = 30°$

$A_o = (0.125)(0.075) = 9.375 \times 10^{-3} \ m^2$

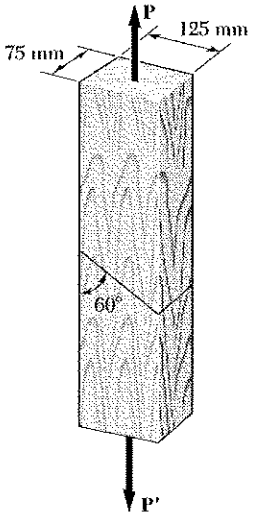
$$\sigma = \frac{P\cos^2\theta}{A_o} = \frac{(6.227 \times 10^3)(\cos 30°)^2}{9.375 \times 10^{-3}}$$

$\sigma = 0.498 \ MPa$

$$\tau = \frac{P\sin 2\theta}{2A_o} = \frac{(6.227 \times 10^3)\sin 60°}{(2)(9.375 \times 10^{-3})}$$

$\tau = 0.288 \ MPa$

Problem 1.30



1.30 Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 525 kPa, determine (*a*) the largest load **P** that can be safely supported, (*b*) the corresponding tensile stress in the splive.

$A_o = (0.125)(0.075) = 9.375 \times 10^{-3} \ m^2$

$\theta = 90° - 60° = 30°$

$$\sigma = \frac{P\cos^2\theta}{A_o}$$

(a) $$P = \frac{\sigma A_o}{\cos^2\theta} = \frac{(525 \times 10^3)(9.375 \times 10^{-3})}{\cos^2 30°} = 6562 N$$

$P = 6.562 \ kN$

(b) $$\tau = \frac{P\sin 2\theta}{2A_o} = \frac{(6562)\sin 60°}{(2)(9.375 \times 10^{-3})}$$

$\tau = 0.303 \ MPa$

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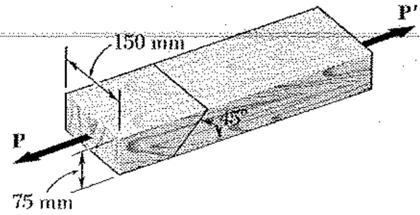
Problem 1.31



1.31 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11$ kN, determine the normal and shearing stresses in the glued splice.

$$\theta = 90° - 45° = 45°$$

$$P = 11 \text{ kN} = 11 \times 10^3 \text{ N}$$

$$A_o = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

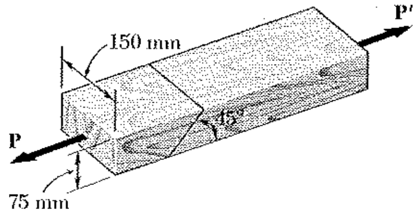
$$\sigma = \frac{P\cos^2\theta}{A_o} = \frac{(11 \times 10^3)\cos^2 45°}{11.25 \times 10^{-3}}$$

$$= 489 \times 10^3 \text{ Pa} \qquad \sigma = 489 \text{ kPA} ◄$$

$$\tau = \frac{P\sin 2\theta}{2A_o} = \frac{(11\times10^3)(\sin 90°)}{(2)(11.25\times10^{-3})}$$

$$= 4.89 \times 10^3 \text{ Pa} \qquad \tau = 489 \text{ kPA} ◄$$

Problem 1.32



1.32 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 560 kPa, determine (*a*) the largest load **P** that can be safely applied, (*b*) the corresponding shearing stress in the splice.

$$\theta = 90° - 45° = 45°$$

$$A_o = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

$$\sigma = 560 \text{ kPa} = 560 \times 10^3 \text{ Pa}$$

$$\sigma = \frac{P\cos^2\theta}{A_o}$$

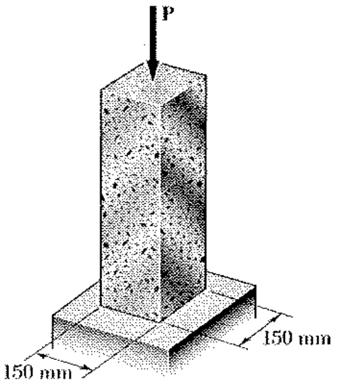
(a) $$P = \frac{\sigma A_o}{\cos^2\theta} = \frac{(560\times10^3)(11.25\times10^{-3})}{\cos^2 45°}$$

$$= 12.60 \times 10^3 \text{ N} \qquad P = 12.60 \text{ kN} ◄$$

(b) $$\tau = \frac{P\sin\theta\cos\theta}{A_o} = \frac{(12.60\times10^3)(\sin 45°)(\cos 45°)}{11.25\times10^{-3}}$$

$$= 560 \times 10^3 \text{ Pa} \qquad \tau = 560 \text{ kPa} ◄$$

Problem 1.33



1.33 A centric load **P** is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 17 MPa, determine (*a*) the magnitude of **P**, (*b*) the orientation of the surface on which the maximum shearing stress occurs, (*c*) the normal stress exerted on the surface, (*d*) the maximum value of the normal stress in the block.

$A_o = (0.15)(0.15) = 0.0225\ m^2$ $\tau_{max} = 17$ MPa

$\theta = 45°$ for plane of τ_{max}

(a) $\tau_{max} = \frac{|P|}{2A_o}$ ∴ $|P| = 2A_o \tau_{max} = (2)(0.0225)(17\times10^6)$

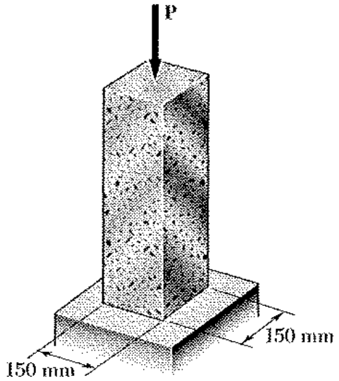
$= 765$ kN ◄

(b) $\sin 2\theta = 1$ $2\theta = 90°$ $\theta = 45°$ ◄

(c) $\sigma_{45} = \frac{P}{A_o}\cos^2 45° = \frac{P}{2A_o} = \frac{-765\times10^3}{2(0.0225)} = -17$ MPa ◄

(d) $\sigma_{max} = \frac{P}{A_o} = -\frac{765\times10^3}{0.0225} = -34$ MPa ◄

Problem 1.34



1.34 A 960-kN load **P** is applied to the granite block shown. Determine the resulting maximum value of (*a*) the normal stress, (*b*) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.

$A_o = (0.15)(0.15) = 0.0225\ m^2$

$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{-960\times10^3}{0.0225}\cos^2\theta = -42.67\times10^6\cos^2\theta$

(a) max tensile stress = 0 at $\theta = 90°$

max. compressive stress = 42.7 MPa ◄

at $\theta = 0°$

(b) $\tau_{max} = \frac{P}{2A_o} = \frac{960\times10^3}{2(0.0225)} = 21.3$ MPa ◄

at $\theta = 45°$

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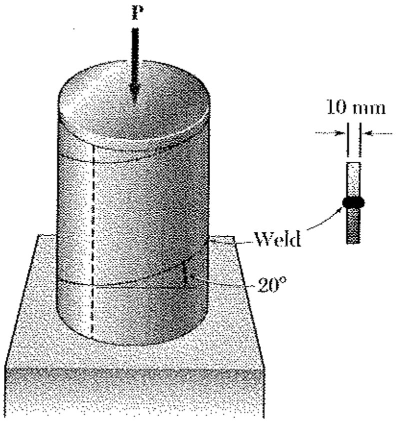
Problem 1.35



1.35 A steel pipe of 400-mm outer diameter is fabricated from 10-mm-thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in the directions respectively normal and tangential to the weld are $\sigma = 60$ MPa and $\tau = 36$ MPa, determine the magnitude P of the largest axial force that can be applied to the pipe.

$d_o = 0.400$ m $\qquad r_o = \frac{1}{2} d_o = 0.200$ m

$r_i = r_o - t = 0.200 - 0.010 = 0.190$ m

$A_o = \pi(r_o^2 - r_i^2) = \pi(0.200^2 - 0.190^2)$

12.25×10^{-3} m^2

$\theta = 20°$

Based on $|\sigma| = 60$ MPa: $\qquad \sigma = \frac{P}{A_o}\cos^2\theta$

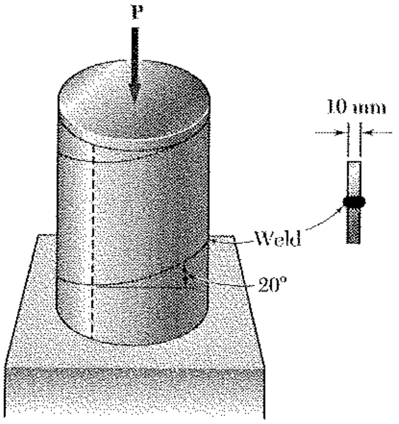
$$P = \frac{A_o\sigma}{\cos^2\theta} = \frac{(12.25 \times 10^{-3})(60 \times 10^6)}{\cos^2 20°} = 833 \times 10^3 \text{ N}$$

Based on $|\tau| = 30$ MPa $\qquad \tau = \frac{P}{2A_o}\sin 2\theta$

$$P = \frac{2A_o\tau}{\sin 2\theta} = \frac{(2)(12.25 \times 10^{-3})(36 \times 10^6)}{\sin 40°} = 1372 \times 10^3 \text{ N}$$

Smaller value is the allowable value of P. $\qquad P = 833$ kN ◄

Problem 1.36



1.36 A steel pipe of 400-mm outer diameter is fabricated from 10-mm-thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that a 300-kN axial force **P** is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

$d_o = 0.400$ m $\qquad r_o = \frac{1}{2} d_o = 0.200$ m

$r_i = r_o - t = 0.200 - 0.010 = 0.190$ m

$A_o = \pi(r_o^2 - r_i^2) = \pi(0.200^2 - 0.190^2)$

$= 12.25 \times 10^{-3}$ m^2

$\theta = 20°$

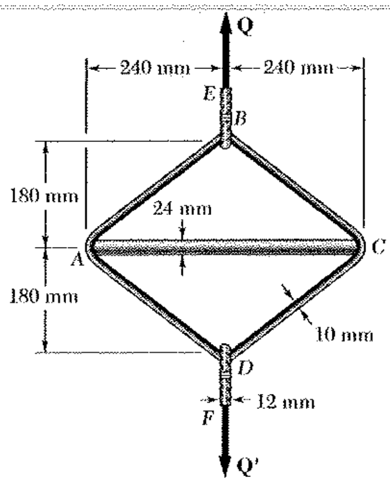
$$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{-300 \times 10^3 \cos^2 20°}{12.25 \times 10^{-3}}$$

$= -21.6 \times 10^6$ Pa $\qquad \sigma = -21.6$ MPa ◄

$$\tau = \frac{P}{2A_o}\sin 2\theta = \frac{-300 \times 10^3 \sin 40°}{(2)(12.25 \times 10^{-3})}$$

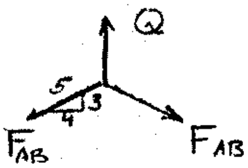
$= -7.87 \times 10^6$ $\qquad \tau = 7.87$ MPa ◄

Problem 1.37



1.37 A steel loop *ABCD* of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod *AC*. Cables *BE* and *DF*, each of 12-mm diameter, are used to apply the load **Q**. Knowing that the ultimate strength of the steel used for the loop and the cables is 480 MPa, determine the largest load **Q** that can be applied if an overall factor of safety of 3 is desired.

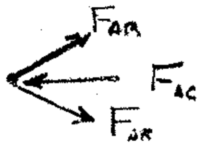
Using joint B as free body and considering symmetry,



$$2 \cdot \frac{3}{5} F_{AB} - Q = 0$$

$$Q = \frac{6}{5} F_{AB}$$

Using joint A as a free body and considering symmetry,



$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore \quad Q = \frac{3}{4} F_{AC}$$

Based on strength of cable BE.

$$Q_U = \sigma_U A = \sigma_U \frac{\pi}{4} d^2 = (480 \times 10^6) \frac{\pi}{4} (0.012)^2 = 54.29 \times 10^3 \text{ N}$$

Based on strength of steel loop.

$$Q_U = \frac{6}{5} F_{AB,U} = \frac{6}{5} \sigma_U A = \frac{6}{5} \sigma_U \frac{\pi}{4} d^2$$

$$= \frac{6}{5} (480 \times 10^6) \frac{\pi}{4} (0.010)^2 = 45.24 \times 10^3 \text{ N}$$

Based on strength of rod AC.

$$Q_U = \frac{3}{4} F_{AC,U} = \frac{3}{4} \sigma_U A = \frac{3}{4} \sigma_U \frac{\pi}{4} d^2$$

$$= \frac{3}{4} (260 \times 10^6) \frac{\pi}{4} (0.024)^2 = 88.22 \times 10^3 \text{ N}$$

Actual ultimate load Q_U is the smallest. $\therefore \quad Q_U = 45.24 \times 10^3$ N

Allowable load $Q = \frac{Q_U}{F.S.} = \frac{45.24 \times 10^3}{3} = 15.08 \times 10^3$ N

$Q = 15.08$ kN ◄

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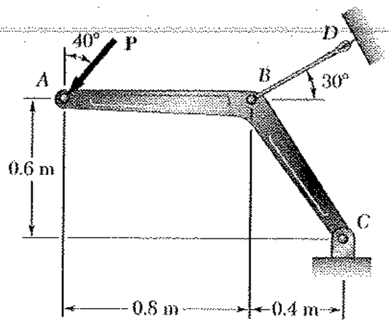
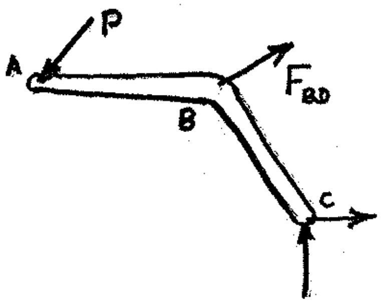
Problem 1.38

1.38 Member *ABC*, which is supported by a pin and bracket at *C* and a cable *BD*, was designed to support the 16-kN load **P** as shown. Knowing that the ultimate load for cable *BD* is 100 kN, determine the factor of safety with respect to cable failure.



Use member ABC as a free body, and note that member BD is a two-force member.



$\stackrel{+}{\circlearrowleft}\Sigma M_C = 0:$

$$(P\cos 40^\circ)(1.2) + (P\sin 40^\circ)(0.6) - (F_{BD}\cos 30^\circ)(0.6) - (F_{BD}\sin 30^\circ)(0.4) = 0$$

$$1.30493\,P - 0.71962\,F_{BD} = 0$$

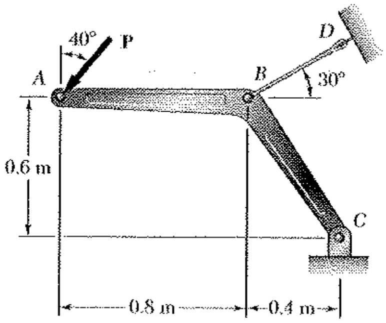
$$F_{BD} = 1.81335\,P = (1.81335)(16\times10^3) = 2.9014\times10^3\ \text{N}$$

$$F_{ult} = 100\times10^3\ \text{N}$$

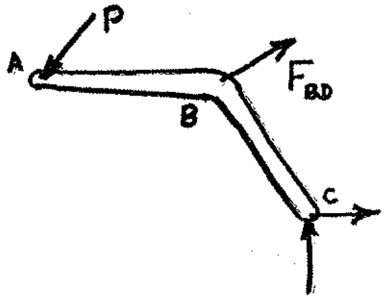
$$F.S. = \frac{F_{ult}}{F_{BD}} = \frac{100\times10^3}{2.9014\times10^3} \qquad F.S. = 3.45 ◄$$

Problem 1.39

1.39 Knowing that the ultimate load for cable *BD* is 100 kN and that a factor of safety of 3.2 with respect to cable failure is required, determine the magnitude of the largest force **P** which can be safely applied as shown to member *ABC*.



Use member ABC as a free body, and note that member BD is a two-force member.



$\stackrel{+}{\circlearrowleft}\Sigma M_C = 0:$

$$(P\cos 40^\circ)(1.2) + (P\sin 40^\circ)(0.6) - (F_{BD}\cos 30^\circ)(0.6) - (F_{BD}\sin 30^\circ)(0.4) = 0$$

$$1.30493\,P - 0.71962\,F_{BD} = 0$$

$$P = 0.55404\,F_{BD}$$

Allowalbe value of F_{BD}.

$$F_{BD} = \frac{F_{ult}}{F.S.} = \frac{100\ \text{kN}}{3.2} = 3.125\ \text{kN}$$

$$P_{all} = (0.55404)(3.125) \qquad P_{all} = 1.732\ \text{kN} ◄$$

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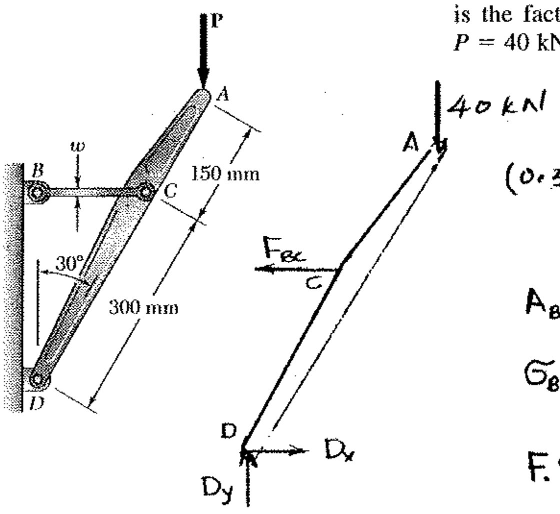
Problem 1.40

1.40 The horizontal link *BC* is 6 mm thick, has a width *w* = 30 mm, and is made of a steel with a 450-MPa ultimate strength in tension. What is the factor of safety if the structure shown is designed to support a load *P* = 40 kN?



$$+\circlearrowleft \Sigma M_C = 0$$

$$(0.3\cos 30°)F_{AB} - (0.45\sin 30°)(40) = 0$$

$$F_{BC} = 34.6\text{ kN}$$

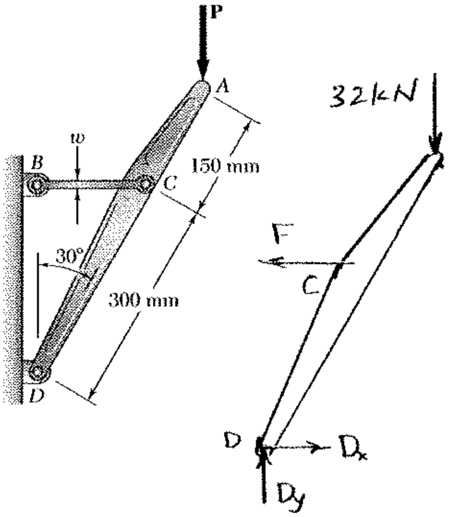
$$A_{BC} = (0.006)(0.03) = 1.8\times10^{-4}\text{ m}^2$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{\sigma_{ult}}{F.S}$$

$$F.S = \frac{A_{BC}\,\sigma_{ult}}{F_{BC}} = \frac{(1.8\times10^{-4})(450\times10^{6})}{34.6\times10^{3}} = 2.34$$

Problem 1.41

1.41 The horizontal link *BC* is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be the width *w* of the link if the structure shown is to be designed to support a load *P* = 32 kN with a factor of safety equal to 3?



$$+\circlearrowleft \Sigma M_C = 0$$

$$(0.3\cos 30°)F_{AB} - (0.45\sin 30°)(32) = 0$$

$$F_{AB} = 27.7\text{ kN}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{F_{AB}}{tw} = \frac{\sigma_{ult}}{F.S.}$$

$$w = \frac{(F.S.)F_{AB}}{t\,\sigma_{ult}} = \frac{(3)(27.7\times10^{3})}{(0.006)(450\times10^{6})}$$

$$w = 0.03078\text{ m} \blacktriangleleft$$

$$= 30.8\text{ mm}$$

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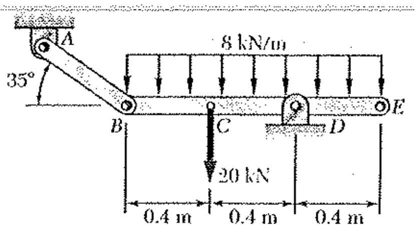
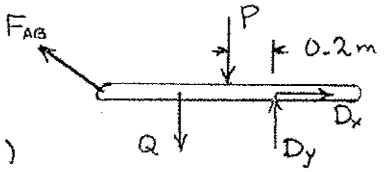
Problem 1.42

1.42 Link *AB* is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area for *AB* for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at *A* and *B*.





$P = (1.2)(8) = 9.6$ kN

$+\circlearrowleft \Sigma M_D = 0:$

$-(0.8)(F_{AB}\sin 35^\circ) + (0.2)(9.6) + (0.4)(20) = 0$

$F_{AB} = 21.619$ kN $= 21.619\times10^3$ N

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sigma_{ult}}{F.S.}$$

$$A_{AB} = \frac{(F.S.)F_{AB}}{\sigma_{ult}} = \frac{(3.50)(21.619\times10^3)}{450\times10^6}$$

$= 168.1\times10^{-6}\ m^2$

$A_{AB} = 168.1\ mm^2$ ◄

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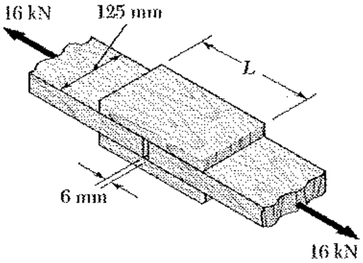
Problem 1.43

1.43 The two wooden members shown, which support a 16-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.5 MPa and the clearance between the members is 6 mm. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.



There are 4 separate areas of glue. Each glue area must transmit 8 kN of shear load.

$$P = 8 \text{ kN} = 8 \times 10^3 \text{ N}$$

Required ultimate load.

$$P_U = (F.S.)P = (2.75)(8 \times 10^3) = 22 \times 10^3 \text{ N}$$

Required length of each glue area.

$$P_U = \tau_U A = \tau_U l w \qquad l = \frac{P_U}{\tau_U w} = \frac{22 \times 10^3}{(2.5 \times 10^6)(0.125)} = 70.4 \times 10^{-3} \text{ m}$$

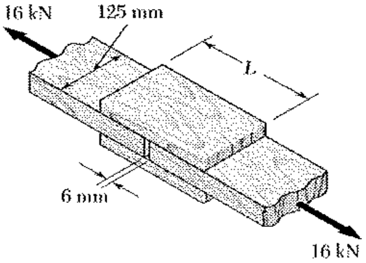
Length of splice: $L = 2l + c = (2)(70.4 \times 10^{-3}) + 0.006 = 0.1468 \times 10^{-3} \text{ m}$

$$L = 146.8 \text{ mm} \blacktriangleleft$$

Problem 1.44

1.44 For the joint and loading of Prob. 1.43, determine the factor of safety, knowing that the length of each splice is $L = 180$ mm.

1.43 The two wooden members shown, which support a 16-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.5 MPa and the clearance between the members is 6 mm. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.



There are 4 separate areas of glue. Each glue area must transmit 8 kN of shear load.

$$P = 8 \text{ kN} = 8 \times 10^3 \text{ N}$$

Length of splice. $L = 2l + c$ where l = length of glue and c = clearance. $l = \frac{1}{2}(L - c) = \frac{1}{2}(0.180 - 0.006) = 0.087 \text{ m}$

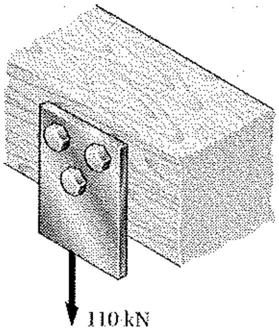
Area of glue. $A = lw = (0.087)(0.125) = 10.875 \times 10^{-3} \text{ m}$

Ulimate load. $P_U = \tau_U A = (2.5 \times 10^6)(10.875 \times 10^{-3}) = 27.1875 \times 10^3 \text{ N}$

Factor of safety. $F.S. = \frac{P_U}{P} = \frac{27.1875 \times 10^3}{8 \times 10^3}$ $\qquad F.S. = 3.40 \blacktriangleleft$

Problem 1.45

1.45 Three 18-mm-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load and that the ultimate shearing stress for the steel used is 360 MPa, determine the factor of safety for this design.



For each bolt, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(18)^2 = 254.47 \text{ mm}^2$

$= 254.47 \times 10^{-6} \text{ m}^2$

$P_U = A\tau_U = (254.47\times10^{-6})(360\times10^{6})$

$= 91.609 \times 10^{3} \text{ N}$

For the three bolts, $P_U = (3)(91.609\times10^{3})$

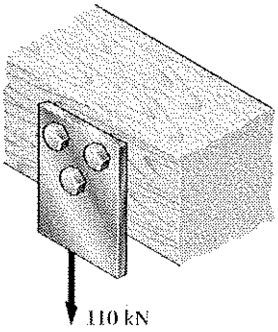
$= 274.83 \times 10^{3} \text{ N}$

Factor of safety.

$F.S. = \frac{P_U}{P} = \frac{274\times10^{3}}{110\times10^{3}} =$ $F.S. = 2.50$ ◀

Problem 1.46

1.46 Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110 kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.



For each bolt, $P = \frac{110}{3} = 36.667 \text{ kN}$

Required $P_U = (F.S.)P = (3.35)(36.667) = 122.83 \text{ kN}$

$\tau_U = \frac{P_U}{A} = \frac{P_U}{\frac{\pi}{4}d^2} = \frac{4P_U}{\pi d^2}$

$d = \sqrt{\frac{4P_U}{\pi \tau_U}} = \sqrt{\frac{(4)(122.83\times10^{3})}{\pi(360\times10^{6})}} = 20.8\times10^{-3} \text{ m}$

$d = 20.8 \text{ mm}$ ◀

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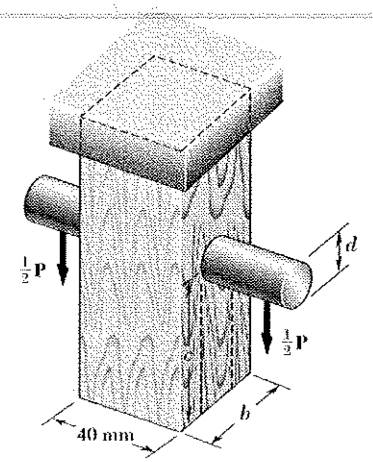
Problem 1.47

1.47 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

1.48 For the support of Prob. 1.47, knowing that the diameter of the pin is $d = 16$ mm and that the magnitude of the load is $P = 20$ kN, determine (*a*) the factor of safety for the pin, (*b*) the required values of *b* and *c* if the factor of safety for the wooden members is the same as that found in part *a* for the pin.



$P = 20 \text{ kN} = 20 \times 10^3 \text{ N}$

(a) Pin: $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

Double shear $\tau = \frac{P}{2A}$ $\tau_U = \frac{P_U}{2A}$

$$P_U = 2A\tau_U = (2)(201.16 \times 10^{-6})(145 \times 10^6)$$

$$= 58.336 \times 10^3 \text{ N}$$

$$F.S. = \frac{P_U}{P} = \frac{58.336 \times 10^3}{20 \times 10^3} = 2.92 \quad ◄$$

(b) Tension in wood $P_U = 58.336 \times 10^3$ N for same F.S.

$$\sigma_U = \frac{P_U}{A} = \frac{P_U}{w(b-d)} \quad \text{where } w = 40 \text{ mm} = 0.040 \text{ m}$$

$$b = d + \frac{P_U}{w\sigma_U} = 0.016 + \frac{58.336 \times 10^3}{(0.040)(60 \times 10^6)} = 40.3 \times 10^3 \text{ m}$$

$$b = 40.3 \text{ mm} \quad ◄$$

Shear in wood $P_U = 58.336 \times 10^3$ N for same F.S.

Double shear; each area is $A = wc$

$$\tau_U = \frac{P_U}{2A} = \frac{P_U}{2wc}$$

$$c = \frac{P_U}{2w\tau_U} = \frac{58.336 \times 10^3}{(2)(0.040)(7.5 \times 10^6)} = 97.2 \times 10^{-3} \text{ m}$$

$$c = 97.2 \text{ mm} \quad ◄$$

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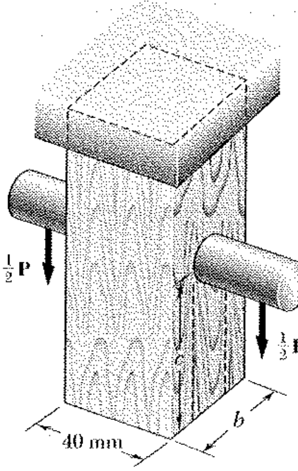
Problem 1.48

1.48 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.



Based on double shear in pin

$$P_U = 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U$$

$$= \frac{\pi}{4}(2)(0.012)^2(145\times10^6) = 32.80\times10^3\ N$$

Based on tension in wood

$$P_U = A\sigma_U = w(b-d)\sigma_U$$

$$=(0.040)(0.040-0.012)(60\times10^6)$$

$$= 67.2\times10^3\ N$$

Based on double shear in the wood

$$P_U = 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5\times10^6)$$

$$= 33.0\times10^3\ N$$

Use smallest $\quad P_U = 32.8\times10^3\ N$

Allowable $\quad P = \frac{P_U}{F.S.} = \frac{32.8\times10^3}{3.2} = 10.25\times10^3\ N$

10.25 kN

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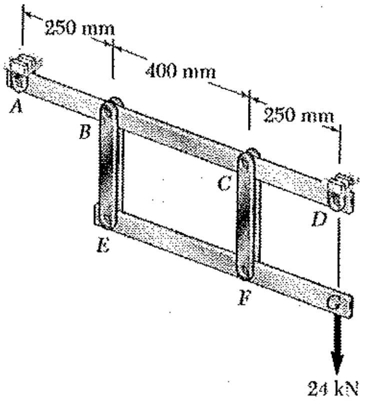
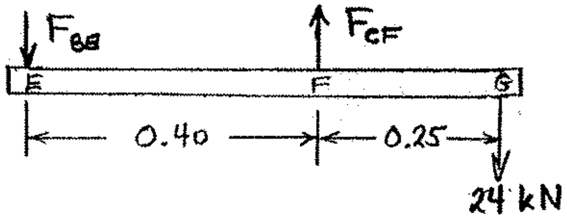
Problem 1.49



1.49 Each of the two vertical links *CF* connecting the two horizontal members *AD* and *EG* has a 10 × 40-mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at *C* and *F* has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.

Use member EFG as free body.



$\textcircled{\scriptsize 1}\ \Sigma M_E = 0$

$0.40\ F_{CF} - (0.65)(24\times10^3) = 0$

$F_{CF} = 39\times10^3\ \text{N}$

Based on tension in links CF

$A = (b-d)\,t = (0.040 - 0.02)(0.010) = 200\times10^{-6}\ \text{m}^2$ (one link)

$F_U = 2\sigma_U A = (2)(400\times10^6)(200\times10^{-6}) = 160.0\times10^3\ \text{N}$

Based on double shear in pins

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}\ \text{m}^2$

$F_U = 2\tau_U A = (2)(150\times10^6)(314.16\times10^{-6}) = 94.248\times10^3\ \text{N}$

Actual F_U is smaller value, i.e. $F_U = 94.248\times10^3\ \text{N}$

Factor of safety $\quad F.S. = \dfrac{F_U}{F_{CF}} = \dfrac{94.248\times10^3}{39\times10^3} = 2.42$ ◄

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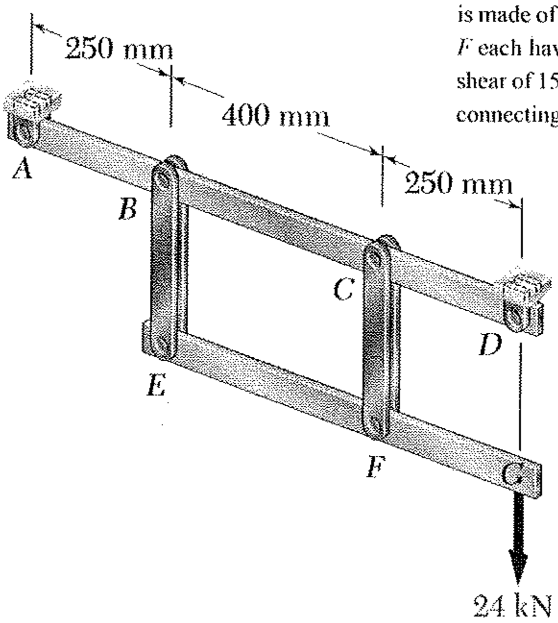
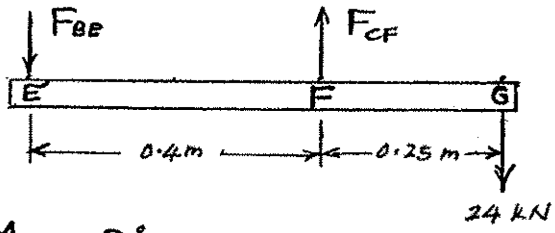
Problem 1.50

1.50 Solve Prob. 1.49, assuming that the pins at *C* and *F* have been replaced by pins with a 30 mm diameter.

1.49 Each of the two vertical links CF connecting the two horizontal members *AD* and *EG* has a uniform rectangular cross section 10 mm thick and 40 mm wide, and is made of a steel with an ultimate strength in tension of 400 MPa. The pins at *C* and *F* each have a 20 mm diameter and are made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.



Use member EFG as free body.



$$\circlearrowleft \Sigma M_E = 0:$$

$$0.4\,F_{CF} - (0.65)(24\times10^{3}) = 0$$

$$F_{CF} = 39\ \text{kN}$$

Failure by tension in links CF. (2 parallel links)

Net section area for 1 link: $A = (b-d)t = (0.04-0.02)(0.01) = 200\times10^{-6}\ m^2$

$$F_U = 2A\sigma_U = (2)(400\times10^{6})(200\times10^{-6}) = 160\ kN$$

Failure by double shear in pins.

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.02)^2 = 314.16\times10^{-6}\ m^2$$

$$F_U = 2A\tau_U = (2)(314.16\times10^{-6})(150\times10^{6}) = 94.25\ kN$$

Actual ultimate load is the smaller value. $F_U = 94.25\ kN$

Factor of safety: $F.S. = \frac{F_U}{F_{CF}} = \frac{94.25}{39}$ $F.S. = 2.42$ ◀

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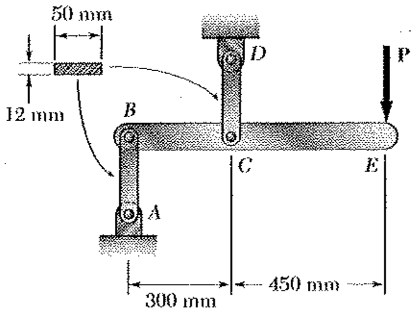
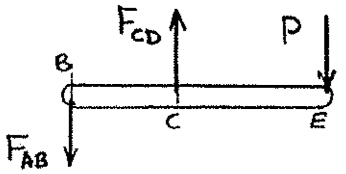
Problem 1.51



1.51 Each of the steel links *AB* and *CD* is connected to a support and to member *BCE* by 25-mm-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 210 MPa for the steel used in the pins and that the ultimate normal stress is 490 MPa for the steel used in the links, determine the allowable load **P** if an overall factor of safety of 3.0 is desired. (Note that the links are not reinforced around the pin holes.)

Use member BCE as free body.



$+\circlearrowleft \Sigma M_B = 0: \quad 0.3F_{CD} - 0.75P = 0$

$P = \frac{2}{5}F_{CD}$

$\circlearrowleft \Sigma M_C = 0: \quad 0.3F_{AB} - 0.45P = 0$

$P = \frac{2}{3}F_{AB}$

Both links have the same area, pin diameter and material, There fore, they have the same ultimate load.

Failure by pin in single shear. $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.025)^2 = 490.9\times10^{-6}\,m^2$

$F_U = \tau_U A = (210\times10^6)(490.9\times10^{-6}) = 103.09\ kN$

Failure by tension in link. $A = (b-d)t = (0.05-0.025)0.012 = 3\times10^{-4}\,m^2$

$F_U = \sigma_U A = (490\times10^6)(3\times10^{-4}) = 147\ kN$

Ultimate load for link and pin is the smaller.

$F_U = 103.09\ kN$

Allowable values of F_{CD} and F_{AB}.

$F_{all} = \frac{F_U}{F.S.} = \frac{103.09}{3.0} = 34.36\ kN$

Allowable load for structure is the smaller of $\frac{2}{3}F_{all}$ and $\frac{2}{5}F_{all}$.

$P = \frac{2}{5}(34.36)$ $\qquad P = 13.7\ kN$ ◀

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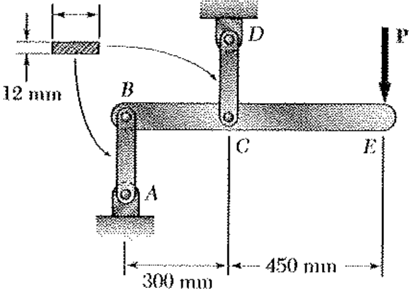
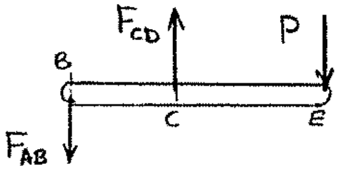
Problem 1.52

1.52 An alternative design is being considered to support member *BCE* of Prob. 1.51, in which link *CD* will be replaced by two links, each of 6×50-mm cross section, causing the pins at *C* and *D* to be in double shear. Assuming that all other specifications remain unchanged, determine the allowable load P if an overall factor of safety of 3.0 is desired.

1.51 Each of the steel links *AB* and *CD* is connected to a support and to member *BCE* by 25-mm-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 210 MPa for the steel used in the pins and that the ultimate normal stress is 490 MPa for the steel used in the links, determine the allowable load **P** if an overall factor of safety of 3.0 is desired. (Note that the links are not reinforced around the pin holes.)



Use member BCE as free body.



$+\circlearrowleft \Sigma M_B = 0: \quad 0.3 F_{CD} - 0.75 P = 0$

$P = \frac{2}{5} F_{CD}$

$+\circlearrowleft \Sigma M_C = 0: \quad 0.3 F_{AB} - 0.45 P = 0$

$P = \frac{2}{3} F_{AB}$

Area of all pins: $A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.025)^2 = 490.9 \times 10^{-6}\ m^2$

Net section area of link AB: $A_{net} = (b-d)t_{AB} = (0.05 - 0.025)(0.012) = 3 \times 10^{-4}\ m^2$

Net section area of the 2 links CD is the same.

Failure by pins A and B in single shear. $(F_{AB})_U = \tau_U A_{pin}$

$(F_{AB})_U = (210 \times 10^6)(490.9 \times 10^{-6}) = 103.09$ kN

Failure by tension in link AB. $(F_{AB})_U = \sigma_U A_{net}$

$(F_{AB})_U = (490 \times 10^6)(3 \times 10^{-4}) = 147$ kN

Ultimate load for link and pins AB is the smaller: $(F_{AB})_U = 103.09$ kN

Corresponding ultimate load, $P_U = \frac{2}{3}(F_{AB})_U = 68.73$ kN

Failure by pins C and in double shear. $(F_{CD})_U = 2\tau_U A_{pin}$

$(F_{CD})_U = (2)(210 \times 10^6)(490.9 \times 10^{-6}) = 206.18$ kN

Failure by tension in links CD. $(F_{CD})_U = \sigma_U A_{net}$

$(F_{CD})_U = (490 \times 10^6)(3 \times 10^{-4}) = 147$ kN

Ultimate load for links and pins CD is the smaller: $(F_{CD})_U = 147$ kN

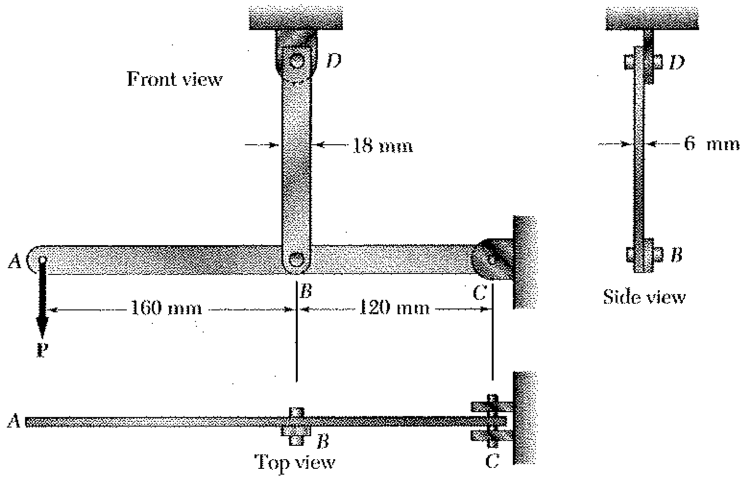
Corresponding ultimate load: $P_U = \frac{2}{5}(F_{CD})_U = 58.8$ kN

Actual ultimate load is the smaller. $P_U = 58.8$ kN

Allowable load P: $P = \frac{P_U}{F.S.} = \frac{58.8}{3.0}$ $\quad P = 19.6$ kN ◀

Problem 1.53

1.53 In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at A. Note that link BD is not reinforced around the pin holes.



Use free body ABC.

$+\circlearrowleft \Sigma M_C = 0:$

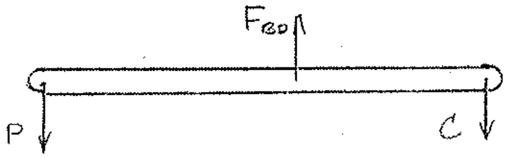
$0.280\,P - 0.120\,F_{BD} = 0$

$P = \frac{3}{7} F_{BD}$ (1)

$+\circlearrowleft \Sigma M_B = 0:$

$0.160\,P - 0.120\,C = 0$

$P = \frac{3}{4} C$ (2)



Tension on net section of link BD.

$$F_{BD} = \sigma A_{net} = \frac{\sigma_U}{F.S.} A_{net}$$

$$= \left(\frac{400\times10^6}{3}\right)(6\times10^{-3})(18-10)(10^{-3})$$

$$= 6.40\times10^3\ N$$

Shear in pins at B and D.

$$F_{BD} = \tau A_{pin} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(10\times10^{-3})^2 = 3.9270\times10^3\ N$$

Smaller value of F_{BD} is 3.9270×10^3 N.

From (1) $P = \left(\frac{3}{7}\right)(3.9270\times10^3) = 1.683\times10^3$ N

Shear in pin at C

$$C = 2\tau A_{pin} = 2\frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (2)\left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(6\times10^{-3})^2 = 2.8274\times10^3\ N$$

From (2) $P = \left(\frac{3}{4}\right)(2.8274\times10^3) = 2.12\times10^3$ N

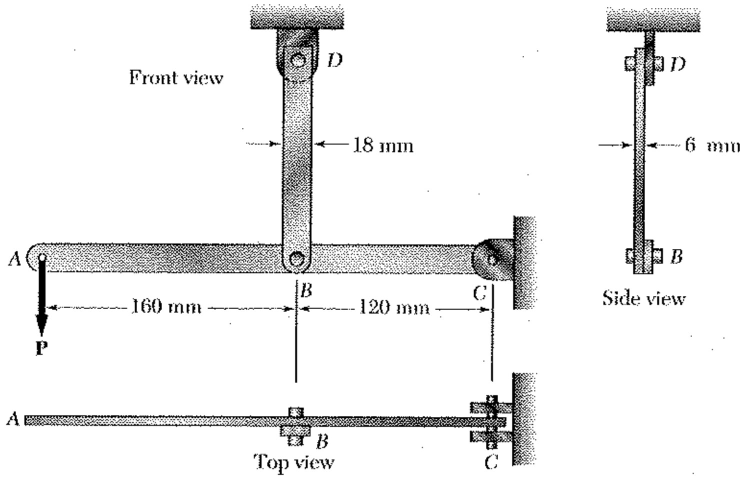
Smaller value of P is allowable value. $P = 1.683\times10^3$ N

$P = 1.683$ kN ◀

Problem 1.54

1.54 Solve Prob. 1.53, assuming that the structure has been redesigned to use 12-mm-diameter pins at *B* and *D* and no other change has been made.

1.53 In the steel structure shown, a 6-mm-diameter pin is used at *C* and 10-mm-diameter pins are used at *B* and *D*. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link *BD*. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at *A*. Note that link *BD* is not reinforced around the pin holes.



Use free body ABC.

$+\circlearrowleft \Sigma M_C = 0:$

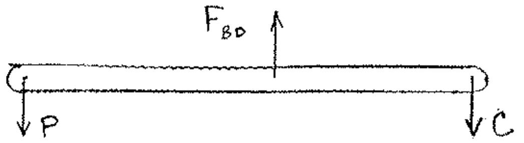
$0.280P - 0.120F_{BD} = 0$

$P = \frac{3}{7}F_{BD}$ (1)

$+\circlearrowleft \Sigma M_B = 0:$

$0.160P - 0.120C = 0$

$P = \frac{3}{4}C$ (2)



Tension on net section of link BD.

$$F_{BD} = \sigma A_{net} = \frac{\sigma_U}{F.S.}A_{net}$$

$$= \left(\frac{400\times10^6}{3}\right)(6\times10^{-3})(18-12)(10^{-3})$$

$$= 4.80\times10^3 \text{ N}$$

Shear in pins at B and D.

$$F_{BD} = \tau A_{pin} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(12\times10^{-3})^2 = 5.6549\times10^3 \text{ N}$$

Smaller value of F_{BD} is 4.80×10^3 N.

From (1), $P = \left(\frac{3}{7}\right)(4.80\times10^3) = 2.06\times10^3$ N

Shear in pin at C.

$$C = 2\tau A_{pin} = 2\frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (2)\left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(6\times10^{-3})^2 = 2.8274\times10^3 \text{ N}$$

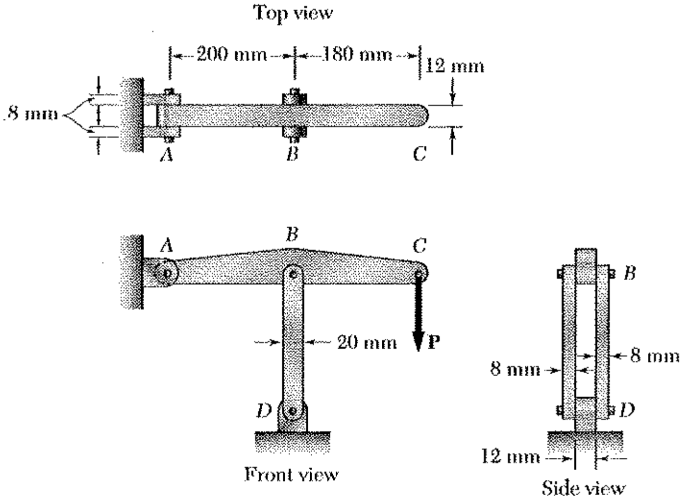
From (2), $P = \left(\frac{3}{4}\right)(2.8274\times10^3) = 2.12\times10^3$ N

Smaller value of P is the allowable value. $P = 2.06\times10^3$ N

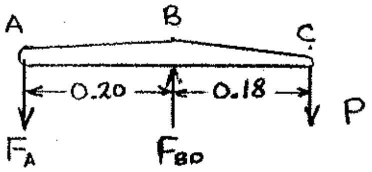
$P = 2.06$ kN ◀

Problem 1.55

1.55 In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.



Statics : Use ABC as free body.



$$\circlearrowleft \Sigma M_B = 0: \quad 0.20\, F_A - 0.18\, P = 0$$

$$P = \frac{10}{9} F_A$$

$$\circlearrowleft \Sigma M_A = 0: \quad 0.20\, F_{BD} - 0.38\, P = 0$$

$$P = \frac{10}{19} F_{BD}$$

Based on double shear in pin A.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.008)^2 = 50.266 \times 10^{-6}\ m^2$$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(50.266\times10^{-6})}{3.0} = 3.351 \times 10^3\ N$$

$$P = \frac{10}{9} F_A = 3.72 \times 10^3\ N$$

Based on double shear in pins at B and D.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ m^2$$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(113.10\times10^{-6})}{3.0} = 7.54 \times 10^3\ N$$

$$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3\ N$$

Based on compression in links BD.

For one link $A = (0.020)(0.008) = 160 \times 10^{-6}\ m^2$

$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250\times10^6)(160\times10^{-6})}{3.0} = 26.7 \times 10^3\ N$$

$$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3\ N$$

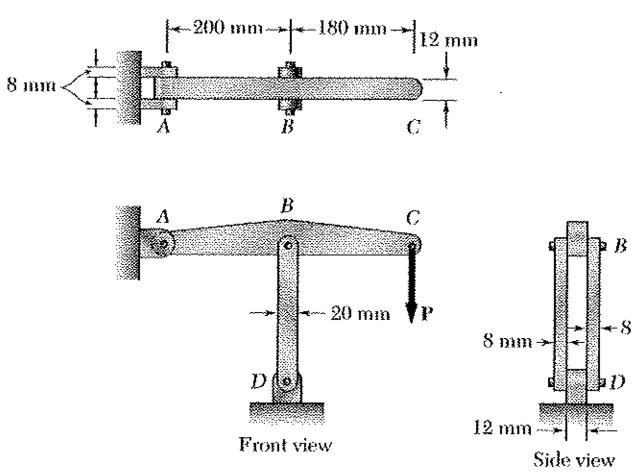
Allowable value of P is smallest. ∴ $P = 3.72 \times 10^3\ N$

$P = 3.72\ kN$ ◄

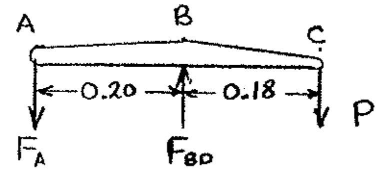
Problem 1.56

1.56 In an alternative design for the structure of Prob. 1.55, a pin of 10-mm-diameter is to be used at *A*. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

1.55 In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.



Statics: Use ABC as free body.



$+\circlearrowleft \Sigma M_B = 0: \quad 0.20\, F_A - 0.18\, P = 0$

$P = \frac{10}{9} F_A$

$+\circlearrowleft \Sigma M_A = 0: \quad 0.20\, F_{BD} - 0.38\, P = 0$

$P = \frac{10}{19} F_{BD}$

Based on double shear in pin A.

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\ m^2$

$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(78.54\times10^{-6})}{3.0} = 5.236\times10^3\ N$

$P = \frac{10}{9} F_A = 5.82\times10^3\ N$

Based on double shear in pins at B and D.

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10\times10^{-6}\ m^2$

$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(113.10\times10^{-6})}{3.0} = 7.54\times10^3\ N$

$P = \frac{10}{19} F_{BD} = 3.97\times10^3\ N$

Based on compression in links BD.

For one link $A = (0.020)(0.008) = 160\times10^{-6}\ m^2$

$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250\times10^6)(160\times10^{-6})}{3.0} = 26.7\times10^3\ N$

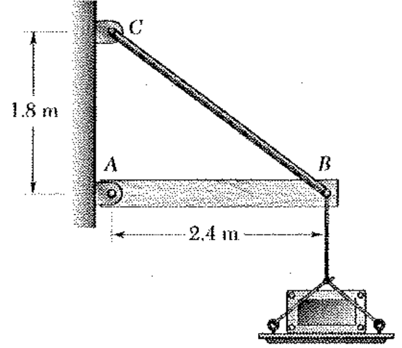
$P = \frac{10}{19} F_{BD} = 14.04\times10^3\ N$

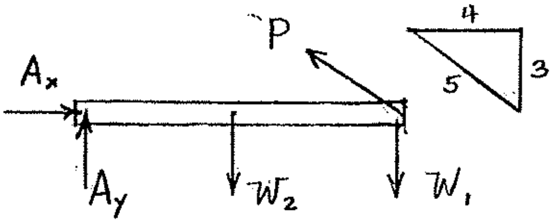
Allowable value of P is smallest. ∴ $P = 3.97\times10^3\ N$

$P = 3.97\ kN$ ◄

Problem 1.57

*1.57 A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (*a*) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_D = 1.25$ and $\gamma_L = 1.6$, determine the largest load that can be safely placed on the platform. (*b*) What is the corresponding conventional factor of safety for rod BC?





$\circlearrowleft + \Sigma M_A = 0: \quad (2.4)\frac{3}{5}P - 2.4\,W_1 - 1.2\,W_2 \quad \therefore \quad P = \frac{5}{3}W_1 + \frac{5}{6}W_2$

For dead loading, $W_1 = (40)(9.81) = 392.4 \text{ N}$

$W_2 = (50)(9.81) = 490.5 \text{ N}$

$P_D = (\frac{5}{3})(392.4) + (\frac{5}{6})(490.5) = 1.0628 \times 10^3 \text{ N}$

For live loading, $W_1 = mg \quad W_2 = 0$

$P_L = \frac{5}{3}mg \quad$ from which $\quad m = \frac{3}{5}\frac{P_L}{g}$

Design criterion.

$\gamma_D P_D + \gamma_L P_L = \phi P_U$

$P_L = \frac{\phi P_U - \gamma_D P_D}{\gamma_L} = \frac{(0.90)(12 \times 10^3) - (1.25)(1.0628 \times 10^{-3})}{1.6}$

$= 5.920 \times 10^3 \text{ N}$

(a) Allowable load. $m = \frac{3}{5}\,\frac{5.92 \times 10^3}{9.81}$ $\qquad m = 362 \text{ kg}$ ◄

Conventional factor safety.

$P = P_D + P_L = 1.0628 \times 10^3 + 5.920 \times 10^3 = 6.983 \times 10^3 \text{ N}$

(b) $\text{F.S.} = \frac{P_U}{P} = \frac{12 \times 10^3}{6.983 \times 10^3}$ $\qquad \text{F.S.} = 1.718$ ◄

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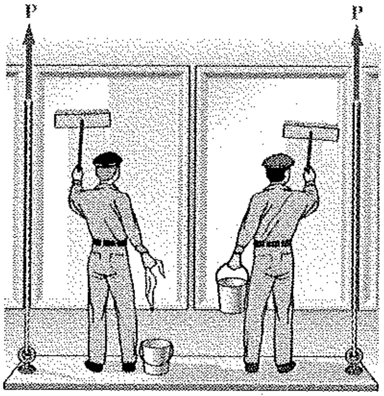
Problem 1.58



***1.58** The Load and Resistance Factor Design method is to be used to select the two cables that will raise and lower a platform supporting two window washers. The platform weighs 72 kg and each of the window washers is assumed to weigh 88 kg with equipment. Since these workers are free to move on the platform, 75% of their total weight and the weight of their equipment will be used as the design live load of each cable. (*a*) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_D = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (*b*) What is the conventional factor of safety for the selected cables?

$$\gamma_D P_D + \gamma_L P_L = \varphi P_U$$

$$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\varphi}$$

$$= \frac{(1.2)(\frac{1}{2}\times 72) + (1.5)(\frac{3}{4}\times 2\times 88)}{0.85}$$

$$= 283.76 \text{ kg} = 2.78 \text{ kN}$$ ◄

Conventional factor of safety

$$P = P_D + P_L = \tfrac{1}{2}\times 72 + 0.75\times 2\times 88 = 168 \text{ kg} = 1.648 \text{ kN}.$$

$$\text{F.S.} = \frac{P_U}{P} = \frac{2.784}{1.648} = 1.69$$ ◄

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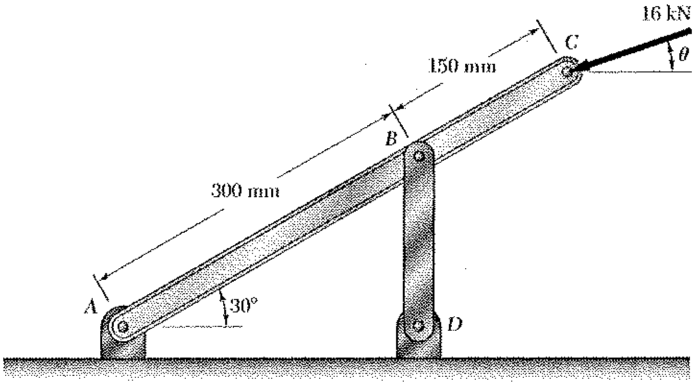
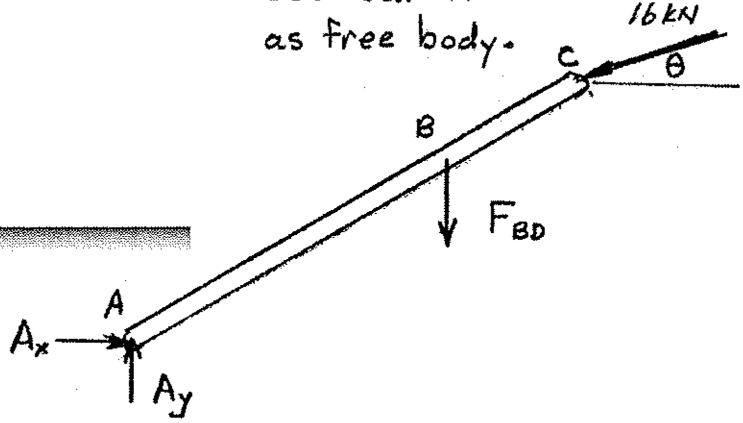
Problem 1.59

1.59 Link *BD* consists of a single bar 24 mm wide and 12 mm thick. Knowing that each pin has a 9 mm diameter, determine the maximum value of the average normal stress in link *BD* if (*a*) $\theta = 0$, (*b*) $\theta = 90°$.



Use bar ABC as free body.



(a) $\theta = 0$.

$+\circlearrowleft \Sigma M_A = 0: \quad (0.45 \sin 30°)(16) - (0.3 \cos 30°) F_{BD} = 0$

$F_{BD} = 13.856$ kN (tension)

Area for tension loading: $A = (b - d)t = (24 - 9)(12) = 180\ mm^2$

Stress: $\sigma = \frac{F_{BD}}{A} = \frac{13.856 \times 10^3}{180 \times 10^{-6}}$ $\sigma = 77\ MPa$ ◄

(b) $\theta = 90°$.

$+\circlearrowleft \Sigma M_A = 0: \quad -(0.45 \cos 30°)(16) - (0.3 \cos 30°) F_{BD} = 0$

$F_{BD} = -24$ kN i.e. compression.

Area for compression loading: $A = bt = (24)(12) = 288\ mm^2$

Stress: $\sigma = \frac{F_{BD}}{A} = \frac{-24000}{288 \times 10^{-6}}$

$\sigma = -83.3$ kN ◄

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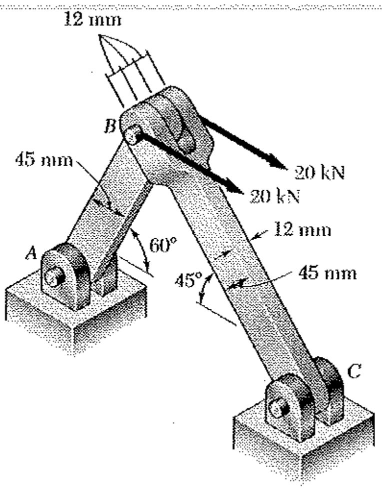
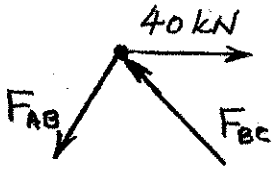
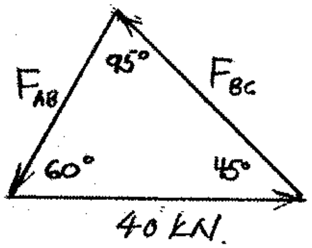
Problem 1.60

1.60 Two horizontal 20 kN forces are applied to pin *B* of the assembly shown. Knowing that a pin of 20 mm diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.



Use joint B as free body.





Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{40}{\sin 95°}$$

$F_{AB} = 28.4\ kN$ $\qquad$ $F_{BC} = 34.8\ kN$

Link AB is a tension member

Minimum section at pin $A_{net} = (0.045 - 0.02)(0.012) = 300 \times 10^{-6}\ m^2$

(a) Stress in AB $\sigma_{AB} = \frac{F_{AB}}{A_{net}} = \frac{28.4}{300 \times 10^{-6}} = 94.7\ MPa$ ◀

Link BC is a compression member

Cross sectional area is $A = (0.045)(0.012) = 540 \times 10^{-6}\ m^2$

(b) Stress in BC $\sigma_{BC} = \frac{-F_{BC}}{A} = \frac{34.8}{540 \times 10^{-6}} = -64.4\ MPa$ ◀

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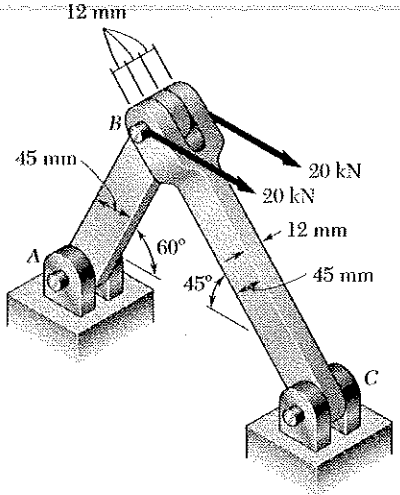
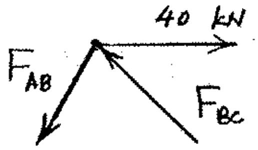
Problem 1.61

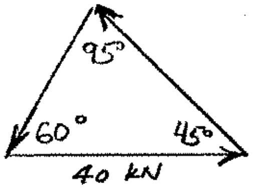
1.61 For the assembly and loading of Prob. 1.60, determine (*a*) the average shearing stress in the pin at *C*, (*b*) the average bearing stress at *C* in member *BC*, (*c*) the average bearing stress at *B* in member *BC*.



1.60 Two horizontal 20-kN forces are applied to pin *B* of the assembly shown. Knowing that a pin of 20-mm diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

Use joint B as free body.





Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{40}{\sin 95^\circ} \qquad F_{BC} = 34.77 \text{ kN}$$

(a) Shearing stress in pin at C. $\tau = \frac{F_{BC}}{2A_P}$

$$A_P = \frac{\pi}{4} d^2 = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2$$

$$\tau = \frac{34770}{(2)(314.16 \times 10^{-6})} = 55.338 \times 10^6 \qquad \tau = 55.3 \text{ MPa}$$ ◄

(b) Bearing stress at C in member BC. $\sigma_b = \frac{F_{BC}}{A}$

$$A = t d = (12)(20) = 240 \text{ mm}^2$$

$$\sigma_b = \frac{34770}{240 \times 10^{-6}} = 144.875 \times 10^6 \qquad \sigma_b = 144.9 \text{ MPa}$$ ◄

(c) Bearing stress at B in member BC. $\sigma_b = \frac{F_{BC}}{A}$

$$A = 2 t d = 2(12)(20) = 480 \text{ mm}^2$$

$$\sigma_b = \frac{34770}{480 \times 10^{-6}} = 72.437 \times 10^6 \qquad \sigma_b = 72.4 \text{ MPa}$$ ◄

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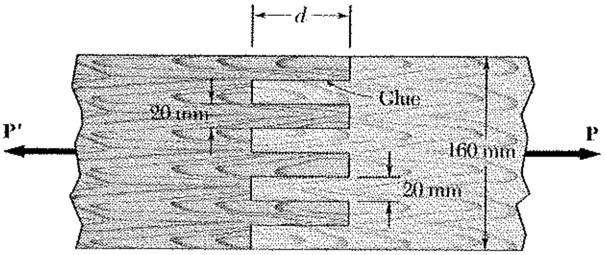
Problem 1.62

1.62 Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length d of the cuts if the joint is to withstand an axial load of magnitude $P = 7.6$ kN.



Seven surfaces carry the total load $P = 7.6 \text{ kN} = 7.6 \times 10^3$.

Let $t = 22$ mm.

Each glue area is $A = dt$

$$\tau = \frac{P}{7A} \qquad A = \frac{P}{7\tau} = \frac{7.6 \times 10^3}{(7)(820 \times 10^3)} = 1.32404 \times 10^{-3} \text{ m}^2$$

$$= 1.32404 \times 10^3 \text{ mm}^2$$

$$d = \frac{A}{t} = \frac{1.32404 \times 10^3}{22} = 60.2$$

$d = 60.2$ mm ◄

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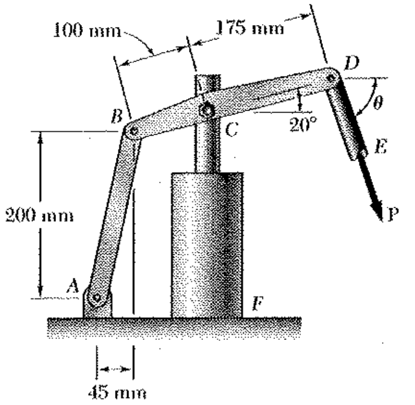
Problem 1.63

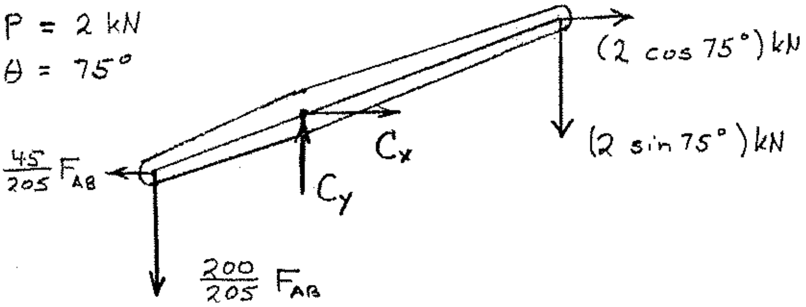
1.63 The hydraulic cylinder *CF*, which partially controls the position of rod *DE*, has been locked in the position shown. Member *BD* is 15 mm thick and is connected to the vertical rod by a 9-mm-diameter bolt. Knowing that $P = 2$ kN and $\theta = 75°$, determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at *C* in member *BD*.



Use member BCD as a free body, and note that AB is a two force member.

$P = 2$ kN

$\theta = 75°$



Length of member AB.

$$l_{AB} = \sqrt{200^2 + 45^2}$$
$$= 205 \text{ mm}$$

$$\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{200}{205} F_{AB}\right)(100 \cos 20°) - \left(\frac{45}{205} F_{AB}\right)(100 \sin 20°) - (2 \cos 75°)(175 \sin 20°) - (2 \sin 75°)(175 \cos 20°) = 0$$

$$84.1696 F_{AB} - 348.668 = 0 \qquad F_{AB} = 4.1424 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - \frac{45}{205}(4.1424) + 2 \cos 75° = 0 \qquad C_x = 0.3917 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - \frac{200}{205}(4.1424) - 2 \sin 75° = 0 \qquad C_y = 5.9732 \text{ kN}$$

$$\text{Reaction at } C: \quad C = \sqrt{C_x^2 + C_y^2} \qquad C = 5.9860 \text{ kN}$$

(a) Shearing stress in bolt (single shear).

$$A_{bolt} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.009)^2 = 63.617 \times 10^{6} \text{ m}^2$$

$$\tau = \frac{C}{A_{bolt}} = \frac{5.9860 \times 10^3}{63.617 \times 10^{-6}} = 94.09 \times 10^6 \text{ Pa} \qquad \tau = 94.1 \text{ MPa} \blacktriangleleft$$

(b) Bearing stress at C in member BD.

$$A_b = dt = (0.009)(0.015) = 135 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{C}{A_b} = \frac{5.9860 \times 10^3}{135 \times 10^{-6}} = 44.34 \times 10^6 \text{ Pa} \qquad \sigma_b = 44.3 \text{ MPa} \blacktriangleleft$$

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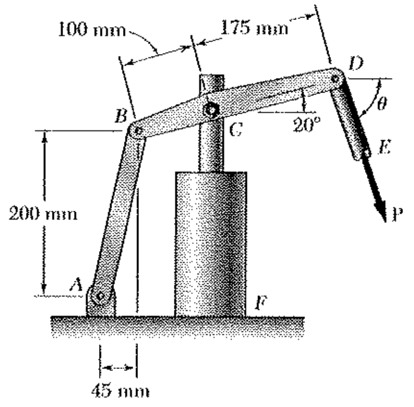
Problem 1.64

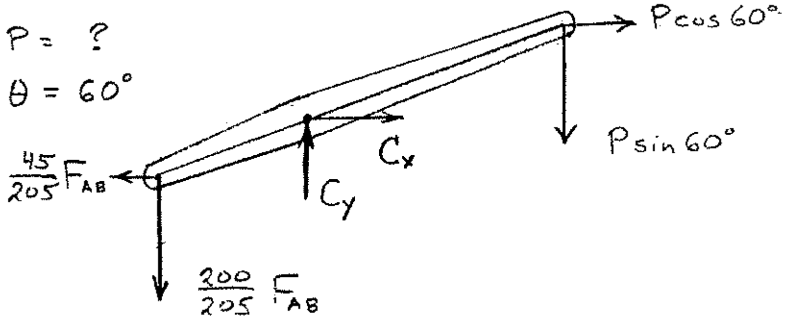
1.64 The hydraulic cylinder *CF*, which partially controls the position of rod *DE*, has been locked in the position shown. Link *AB* has a uniform rectangular cross section of 12 × 25 mm and is connected at *B* to member *BD* by an 8-mm-diameter pin. Knowing that the maximum allowable average shearing stress in the pin is 140 MPa, determine (*a*) the largest force **P** which may be applied at *E* when $\theta = 60°$, (*b*) the corresponding bearing stress at *B* in link *AB*, (*c*) the corresponding maximum value of the normal stress in link *AB*.



Use member BCD as a free body, and note that AB is a two force member.

$P = ?$

$\theta = 60°$



Length of member AB.

$$l_{AB} = \sqrt{200^2 + 45^2}$$

$$= 205 \text{ mm}$$

$$+\circlearrowleft \Sigma M_C = 0: \left(\frac{200}{205} F_{AB}\right)(100 \cos 20°) - \left(\frac{45}{205} F_{AB}\right)(100 \sin 20°)$$

$$-(P \cos 60°)(175 \sin 20°) - (P \sin 60°)(175 \cos 20°) = 0$$

$$84.1696\, F_{AB} - 172.3414\, P = 0 \qquad F_{AB} = 2.0475\, P$$

(a) Allowable load P. Pin at A is in single shear.

$$A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.008)^2 = 50.2655 \times 10^{-6} \text{ m}^2$$

$$\tau = 140 \times 10^6 \text{ Pa}$$

$$\tau = \frac{F_{AB}}{A_{pin}} \qquad 140 \times 10^6 = \frac{2.0475\, P}{50.2655 \times 10^{-6}}$$

$$P = 3.4370 \times 10^3 \text{ N} \qquad P = 3.44 \text{ kN}$$

(b) Bearing stress at B in link AB. $d = 8$ mm, $t = 12$ mm

$$A_b = dt = (0.008)(0.012) = 96 \times 10^{-6} \text{ m}^2$$

$$F_{AB} = (2.0475)(3.4375 \times 10^3) = 7.0383 \times 10^3 \text{ N}$$

$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{7.0383 \times 10^3}{96 \times 10^{-6}} = 73.3 \times 10^6 \text{ Pa} \qquad \sigma_b = 73.3 \text{ MPa}$$

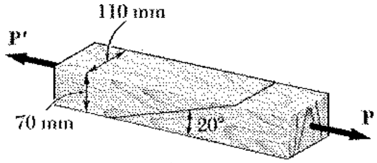
(c) Maximum normal stress in link AB. $b = 25$ mm, $t = 0.012$ mm

$$A_{net} = (b - d)(t) = (0.025 - 0.008)(0.012) = 204 \times 10^{-6} \text{ m}^2$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{net}} = \frac{7.0383 \times 10^3}{204 \times 10^{-6}} = 34.5 \times 10^6 \text{ Pa} \qquad \sigma_{AB} = 34.5 \text{ MPa}$$

Problem 1.65

1.65 Two wooden members of 70 × 110-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 500 kPa, determine the largest axial load **P** that can be safely applied.



$$A_o = (0.070\,\text{m})(0.110\,\text{m}) = 7.7\times10^{-3}\ \text{m}^2$$

$$\theta = 90^\circ - 20^\circ = 70^\circ$$

$$\tau = \frac{P}{A_o}\sin\theta\cos\theta$$

$$P = \frac{A_o\tau}{\sin\theta\cos\theta} = \frac{(7.7\times10^{-3})(500\times10^{3})}{\sin 70^\circ\cos 70^\circ} = 11.98\times10^{3}\ \text{N}$$

$$P = 11.98\ \text{kN}$$ ◄

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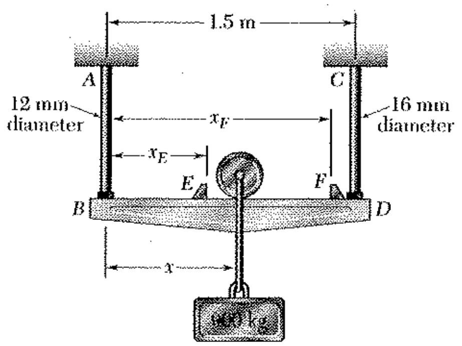
Problem 1.66



1.66 The 900-kg load may be moved along the beam *BD* to any position between stops at *E* and *F*. Knowing that $\sigma_{all} = 42$ MPa for the steel used in rods *AB* and *CD*, determine where the stops should be placed if the permitted motion of the load is to be as large as possible.

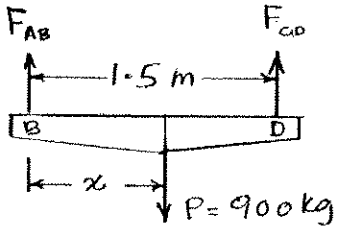
Permitted member forces:

$$AB: \quad (F_{AB})_{max} = \sigma_{all} A_{AB} = (42\times10^6)(\tfrac{\pi}{4})(0.012)^2$$

$$= 4.75 \text{ kN}$$

$$CD: \quad (F_{CD})_{max} = \sigma_{all} A_{CD} = (42\times10^6)(\tfrac{\pi}{4})(0.016)^2$$

$$= 8.44 \text{ kN}$$



Use member BEFD as a free body.

$$P = 900 \text{ kg} = 8.829 \text{ kN}$$

$$+\circlearrowleft \Sigma M_D = 0$$

$$-(1.5)F_{AB} + (1.5 - x_E)P = 0$$

$$1.5 - x_E = \frac{1.5 F_{AB}}{P} = \frac{(1.5)(4.75\times10^3)}{8.829\times10^3}$$

$$= 0.807$$

$$x_E = 0.693 \text{ m}$$

$$+\circlearrowleft \Sigma M_B = 0$$

$$1.5 F_{CD} - xP = 0$$

$$x_F = \frac{1.5 F_{CD}}{P} = \frac{(1.5)(8.44)}{8.829}$$

$$x_F = 1.43 \text{ m}$$

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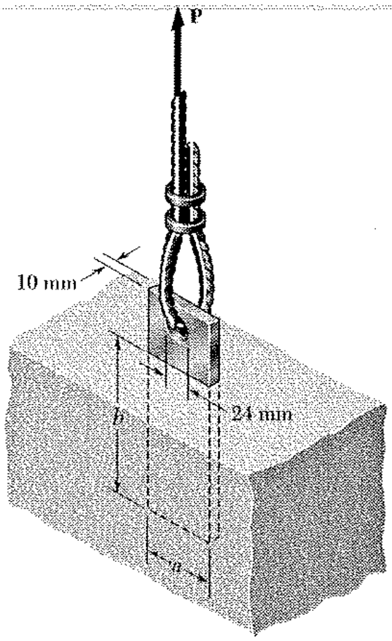
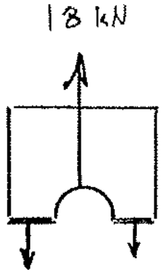
Problem 1.67

1.67 A steel plate 10 mm thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is 24 mm, the ultimate strength of the steel used is 250 MPa, and the ultimate bonding stress between plate and concrete is 2.1 MPa. Knowing that a factor of safety of 3.60 is desired when $P = 18$ kN, determine (*a*) the required width *a* of the plate, (*b*) the minimum depth *b* to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)





(a) Based on tension in the plate.

$$A = (a-d)t$$

$$P_U = \sigma_U A$$

$$F.S. = \frac{P_U}{P} = \frac{\sigma_U (a-d)t}{P}$$

Solving for a,

$$a = d + \frac{(F.S.)P}{\sigma_U t} = 0.024 + \frac{(3.60)(18\times10^{3})}{(250\times10^{6})(0.010)}$$

$$a = 0.04992 \text{ m} \qquad a = 49.9 \text{ mm}$$

(b) Based on shear between plate and concrete slab.

$$A = \text{perimeter} \times \text{depth} = (2a+2t)\,b \qquad \tau_U = 2.1\times10^{6} \text{ Pa}$$

$$P_U = \tau_U A = 2\tau_U (a+t)\,b \qquad F.S. = \frac{P_U}{P}$$

Solving for b,

$$b = \frac{(F.S.)P}{2(a+t)\tau_U} = \frac{(3.60)(18\times10^{3})}{(2)(0.04992+0.010)(2.1\times10^{6})}$$

$$b = 0.25748 \text{ m} \qquad b = 257 \text{ mm}$$

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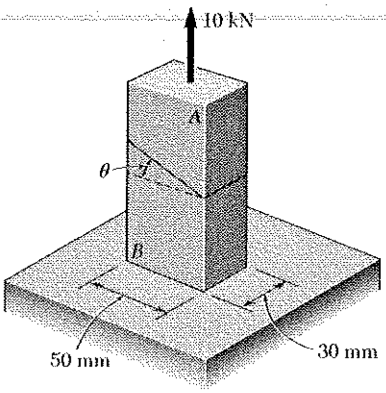
Problem 1.68



1.68 The two portions of member *AB* are glued together along a plane forming an angle *θ* with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine the range of values of *θ* for which the factor of safety of the members is at least 3.0.

$A_o = (0.05)(0.03) = 0.0015\ m^2$

$P = 10\ kN$ $\qquad P_U = (F.S)P = 30\ kN$

Based on tensile stress

$$\sigma_U = \frac{P_U}{A_o}\cos^2\theta$$

$$\cos^2\theta = \frac{\sigma_U A_o}{P_U} = \frac{(17\times10^6)(0.0015)}{30\times10^3} = 0.85$$

$\cos\theta = 0.922$ $\qquad \theta = 22.8°$ $\qquad \theta \geqslant 22.8°$

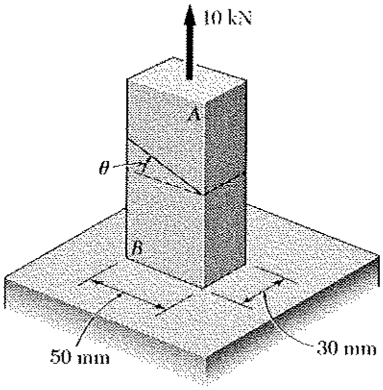
Based on shearing stress $\qquad \tau_U = \frac{P_U}{A_o}\sin\theta\cos\theta = \frac{P_U}{2A_o}\sin 2\theta$

$$\sin 2\theta = \frac{2A_o\tau_U}{P_U} = \frac{(2)(0.0015)(9\times10^6)}{30\times10^3} = 0.9$$

$2\theta = 64.16°$ $\qquad \theta = 32.1°$ $\qquad \theta \leqslant 32.1°$

Hence $\quad 22.8° \leqslant \theta \leqslant 32.1°$ ◄

Problem 1.69



1.69 The two portions of member *AB* are glued together along a plane forming an angle *θ* with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine (*a*) the value of *θ* for which the factor of safety of the member is maximum, (*b*) the corresponding value of the factor of safety. (*Hint*: Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

$A_o = (0.05)(0.03) = 0.0015\ m^2$

At the optimum angle $\quad (F.S.)_\sigma = (F.S.)_\tau$

Normal stress: $\sigma = \frac{P}{A_o}\cos^2\theta \quad \therefore \quad P_{U,\sigma} = \frac{\sigma_U A_o}{\cos^2\theta}$

$$(F.S.)_\sigma = \frac{P_{U,\sigma}}{P} = \frac{\sigma_U A_o}{P\cos^2\theta}$$

Shearing stress: $\tau = \frac{P}{A_o}\sin\theta\cos\theta \quad \therefore \quad P_{U,\tau} = \frac{\tau_U A_o}{\sin\theta\cos\theta}$

$$(F.S.)_\tau = \frac{P_{U,\tau}}{P} = \frac{\tau_U A_o}{P\sin\theta\cos\theta}$$

Equating: $\frac{\sigma_U A_o}{P\cos^2\theta} = \frac{\tau_U A_o}{P\sin\theta\cos\theta}$;

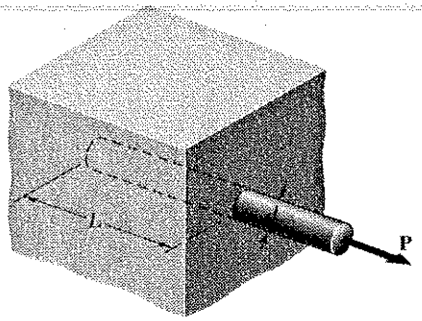
Solving: $\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{\tau_U}{\sigma_U} = \frac{9}{17} = 0.529$ $\qquad$ (a) $\theta_{opt} = 27.9°$ ◄

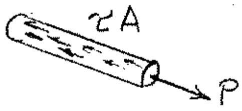
(b) $\quad P_U = \frac{\sigma_U A_o}{\cos^2\theta} = \frac{(17\times10^6)(0.0015)}{\cos^2 27.9°} = 32.65\ kN$

$F.S. = \frac{P_U}{P} = \frac{32.64}{10} = 3.26$ ◄

Problem 1.70

1.70 A force **P** is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{all} in the steel, and the average allowable bond stress τ_{all} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar.)





For shear, $A = \pi d L$

$P = \tau_{all} A = \tau_{all} \pi d \cdot L$

For tension, $A = \frac{\pi}{4} d^2$

$P = \sigma_{all} A = \sigma_{all} (\frac{\pi}{4} d^2)$

Equating, $\tau_{all} \pi d L = \sigma_{all} \frac{\pi}{4} d^2$

Solving for L,

$L_{min} = \sigma_{all} d / 4 \tau_{all}$ ◀

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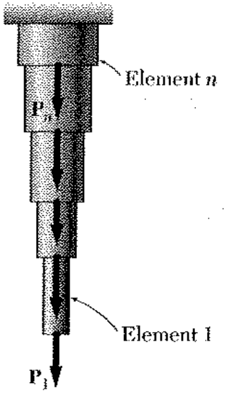
PROBLEM 1.C1



1.C1 A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by $\mathbf{P}_i$, with the magnitude P_i of this load being assumed positive if $\mathbf{P}_i$ is directed downward as shown and negative otherwise. (*a*) Write a computer program to determine the average stress in each element of the rod. (*b*) Use this program to solve Probs. 1.2 and 1.4.

SOLUTION

FORCE IN ELEMENT i:

It is the sum of the forces applied to that element and all lower ones:

$$F_i = \sum_{k=1}^{i} P_k$$

AVERAGE STRESS IN ELEMENT i:

Area $= A_i = \frac{1}{4}\pi d_i^2$ Ave stress $= \frac{F_i}{A_i}$

PROGRAM OUTPUTS

```
      Problem   1.2                  Problem 1.4
   Element  Stress (MPa)          Element Stress (MPa)

      1        81·487                1       35·651
      2       -18·108                2       42·441
```

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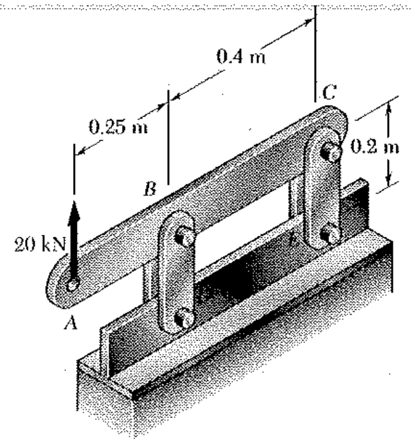
PROBLEM 1.C2

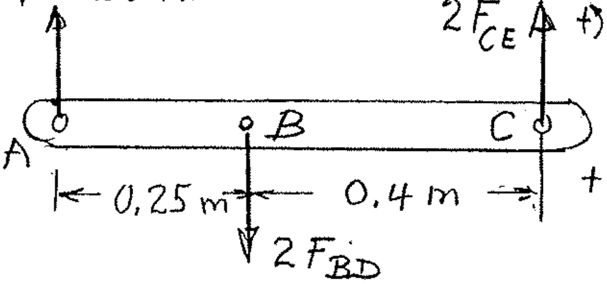


1.C2 A 20-kN load is applied as shown to the horizontal member *ABC*. Member *ABC* has a 10 × 50-mm uniform rectangular cross section and is supported by four vertical links, each of 8 × 36-mm uniform rectangular cross section. Each of the four pins at *A*, *B*, *C*, and *D* has the same diameter *d* and is in double shear. (*a*) Write a computer program to calculate for values of *d* from 10 to 30 mm, using 1-mm increments, (1) the maximum value of the average normal stress in the links connecting pins *B* and *D*, (2) the average normal stress in the links connecting pins *C* and *E*, (3) the average shearing stress in pin *B*, (4) the average shearing stress in pin *C*, (5) the average bearing stress at *B* in member *ABC*, (6) the average bearing stress at *C* in member *ABC*. (*b*) Check your program by comparing the values obtained for *d* = 16 mm with the answers given for Probs. 1.7 and 1.27. (*c*) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively 150 MPa, 90 MPa, and 230 MPa. (*d*) Solve part *c*, assuming that the thickness of member *ABC* has been reduced from 10 to 8 mm.

SOLUTION

FORCES IN LINKS

$P = 20$ kN



F.B. DIAGRAM OF ABC:

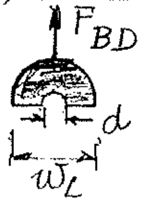
$+\circlearrowleft \sum M_C = 0: \ 2F_{BD}(BC) - P(AC) = 0$

$F_{BD} = P(AC)/2(BC)$ (TENSION)

$+\circlearrowright \sum M_B = 0: \ 2F_{CE}(BC) - P(AB) = 0$

$F_{CE} = P(AB)/2(BC)$ (COMP.)

(1) LINK BD



Thickness $= t_L$

$A_{BD} = t_L(w_L - d)$

$\sigma_{BD} = +F_{BD}/A_{BD}$

(3) PIN B

$\tau_B = F_{BD}/(\pi d^2/4)$

(5) BEARING STRESS AT B

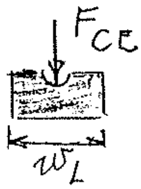
Thickness of member $AC = t_{AC}$

Sig Bear B $= F_{BD}/(d\,t_{AC})$

(6) BEARING STRESS AT C

Sig Bear C $= F_{CE}/(d\,t_{AC})$

(2) LINK CE



Thickness $= t_L$

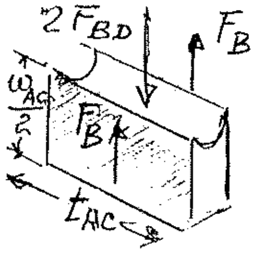
$A_{CE} = t_L w_L$

$\sigma_{CE} = -F_{CE}/A_{CE}$

(4) PIN C

$\tau_C = F_{CE}/(\pi d^2/4)$

SHEARING STRESS IN ABC UNDER PIN B



$F_B = \tau_{AC}\, t_{AC}(w_{AC}/2)$

$\sum F_y = 0: \ 2F_B = 2F_{BD}$

$\tau_{AC} = \dfrac{2F_{BD}}{t_{AC}\, w_{AC}}$

(CONTINUED)

PROBLEM 1.C2 CONTINUED PROGRAM OUTPUTS

INPUT DATA FOR PARTS (a), (b), (c): P = 20 kN, AB = 0.25 m, BC = 0.40 m, AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 10 mm, WAC = 50 mm

```
  d     Sigma BD Sigma CE   Tau B     Tau C  SigBear B SigBear C

 10.00    78.13   -21.70   206.90     79.58   325.00    125.00
 11.00    81.25   -21.70   170.99     65.77   295.45    113.64
 12.00    84.64   -21.70   143.68     55.26   270.83    104.17
 13.00    88.32   -21.70   122.43     47.09   250.00     96.15
 14.00    92.33   -21.70   105.56     40.60   232.14     89.29
 15.00    96.73   -21.70    91.96     35.37   216.67     83.33
 16.00   101.56   -21.70    80.82     31.08   203.12     78.13 ← (b)
 17.00   106.91   -21.70    71.59     27.54   191.18     73.53
 18.00   112.85   -21.70    63.86     24.56   180.56     69.44
 19.00   119.49   -21.70    57.31     22.04   171.05     65.79
 20.00   126.95   -21.70    51.73     19.89   162.50     62.50
 21.00   135.42   -21.70    46.92     18.04   154.76     59.52
 22.00   145.09   -21.70    42.75     16.44   147.73     56.82
 23.00   156.25   -21.70    39.11     15.04   141.30     54.35
 24.00   169.27   -21.70    35.92     13.82   135.42     52.08
 25.00   184.66   -21.70    33.10     12.73   130.00     50.00
 26.00   203.13   -21.70    30.61     11.77   125.00     48.08
 27.00   225.69   -21.70    28.38     10.92   120.37     46.30
 28.00   253.91   -21.70    26.39     10.15   116.07     44.64
 29.00   290.18   -21.70    24.60      9.46   112.07     43.10
 30.00   338.54   -21.70    22.99      8.84   108.33     41.67
```

(c) ANSWER : 16 mm ≤ d ≤ 22 mm ◀ (c)

CHECK: For d = 22 mm, Tau AC = 65 MPa < 90 MPa O.K.

INPUT DATA FOR PART (d): P = 20 kN, AB = 0.25 m, BC = 0.40 m, AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 8 mm, WAC = 50 mm

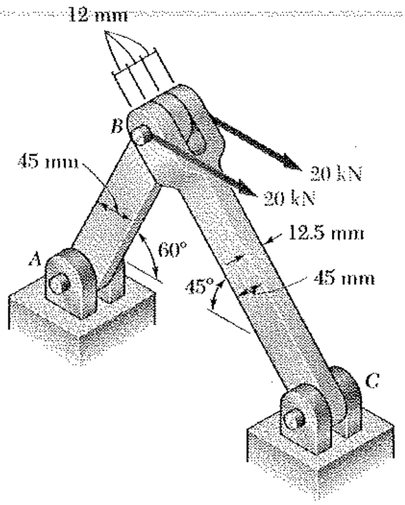
```
  d     Sigma BD Sigma CE   Tau B     Tau C  SigBear B SigBear C

 10.00    78.13   -21.70   206.90     79.58   406.25    156.25
 11.00    81.25   -21.70   170.99     65.77   369.32    142.05
 12.00    84.64   -21.70   143.68     55.26   338.54    130.21
 13.00    88.32   -21.70   122.43     47.09   312.50    120.19
 14.00    92.33   -21.70   105.56     40.60   290.18    111.61
 15.00    96.73   -21.70    91.96     35.37   270.83    104.17
 16.00   101.56   -21.70    80.82     31.08   253.91     97.66
 17.00   106.91   -21.70    71.59     27.54   238.97     91.91
 18.00   112.85   -21.70    63.86     24.56   225.69     86.81
 19.00   119.49   -21.70    57.31     22.04   213.82     82.24
 20.00   126.95   -21.70    51.73     19.89   203.12     78.13
 21.00   135.42   -21.70    46.92     18.04   193.45     74.40
 22.00   145.09   -21.70    42.75     16.44   184.66     71.02
 23.00   156.25   -21.70    39.11     15.04   176.63     67.93
 24.00   169.27   -21.70    35.92     13.82   169.27     65.10
 25.00   184.66   -21.70    33.10     12.73   162.50     62.50
 26.00   203.13   -21.70    30.61     11.77   156.25     60.10
 27.00   225.69   -21.70    28.38     10.92   150.46     57.87
 28.00   253.91   -21.70    26.39     10.15   145.09     55.80
 29.00   290.18   -21.70    24.60      9.46   140.09     53.88
 30.00   338.54   -21.70    22.99      8.84   135.42     52.08
```

(d) ANSWER : 18 mm ≤ d ≤ 22 mm ◀ (d)

CHECK: For d = 22 mm, Tau AC = 81.25 MPa < 90 MPa O.K.

PROBLEM 1.C3

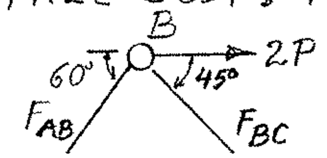


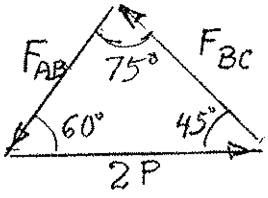
1.C3 Two horizontal 20 kN forces are applied to pin *B* of the assembly shown. Each of the three pins at *A*, *B*, and *C* has the same diameter *d* and is in double shear. (*a*) Write a computer program to calculate for values of *d* from 12.5 mm to 37.5 mm, using 1.25-mm increments, (1) the maximum value of the average normal stress in member *AB*, (2) the average normal stress in member *BC*, (3) the average shearing stress in pin *A*, (4) the average shearing stress in pin *C*, (5) the average bearing stress at *A* in member *AB*, (6) the average bearing stress at *C* in member *BC*, (7) the average bearing stress at *B* in member *BC*. (*b*) Check your program by comparing the values obtained for $d = 20$ mm with the answers given for Probs. 1.60 and 1.61. (*c*) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 150 MPa, 90 MPa, and 250 MPa. (*d*) Solve part *c*, assuming that a new design is being investigated in which the thickness and width of the two members are changed, respectively, from 12.5 to 8 mm and from 45 mm to 60 mm.

SOLUTION

FORCES IN MEMBERS AB AND BC

FREE BODY: PIN B





FROM FORCE TRIANGLE:

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{2P}{\sin 75°}$$

$$F_{AB} = 2P(\sin 45°/\sin 75°)$$

$$F_{BC} = 2P(\sin 60°/\sin 75°)$$

(1) MAX. AVE. STRESS IN AB

Width = w

Thickness = t

$A_{AB} = (w - d)t$

$\sigma_{AB} = F_{AB}/A_{AB}$

(3) PIN A

$\tau_A = (F_{AB}/2)/(\pi d^2/4)$

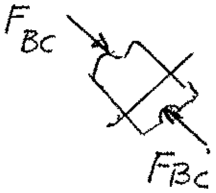
(5) BEARING STRESS AT A

Sig Bear A = F_{AB}/dt

(7) BEARING STRESS AT B IN MEMBER BC

Sig Bear B = $F_{BC}/2dt$

(2) AVE. STRESS IN BC



$A_{BC} = wt$

$\sigma_{BC} = F_{BC}/A_{BC}$

(4) PIN C

$\tau_C = (F_{BC}/2)/(\pi d^2/4)$

(6) BEARING STRESS AT C

Sig Bear C = F_{BC}/dt

(CONTINUED)

PROBLEM 1.C3 CONTINUED **PROGRAM OUTPUTS**

INPUT DATA FOR PARTS (a), (b), (c): P = 20 kN w = 45 mm t = 12 mm

D mm	SIGAB MPa	SIGBC MPa	TAUA MPa	TAUC MPa	SIGBRGA MPa	SIGBRGC MPa	SIGBRGB MPa
12.5	77.651	-68.687	125.537	157.420	201.890	247.275	123.641
17.5	91.773	-68.687	65.578	80.320	144.216	176.622	88.311
20	100.950	-68.687	50.209	61.490	126.185	154.544	77.272 ← (b)
27.5	144.216	-68.687	26.560	32.524	91.773	112.395	56.201
37.5	336.497	-68.687	14.280	17.493	67.302	82.423	41.211

(c) ANSWER: $17.5\text{ mm} \le d \le 27.5\text{ mm}$

INPUT DATA FOR PART (d): P = 20 kN w = 60 mm t = 8 mm

D mm	SIGAB MPa	SIGBC MPa	TAUA MPa	TAUC MPa	SIGBRA MPa	SIGBRC MPa	SIGBRB MPa
12.5	88.552	-85.857	128.537	157.420	336.497	412.128	206.064
20	105.156	-85.857	50.209	61.490	210.311	257.577	128.792
25	120.180	-85.857	31.131	39.357	168.252	206.064	103.032
31.25	146.305	-85.857	20.568	28.187	134.597	164.853	82.423
37.5	186.944	-85.857	14.280	17.493	112.168	137.376	68.688

(d) ANSWER: $21.25\text{ mm} \le d \le 31.25\text{ mm}$

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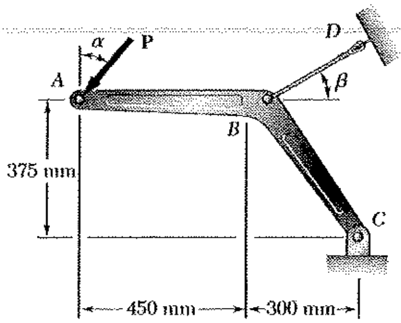
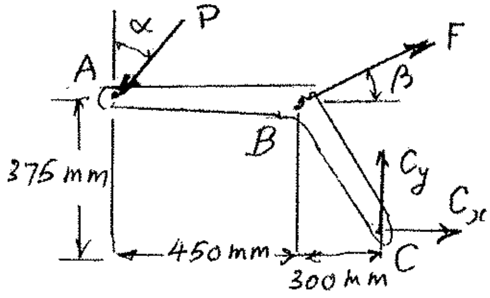
PROBLEM 1.C4



1.C4 A 16 kN force **P** forming an angle α with the vertical is applied as shown to member ABC, which is supported by a pin and bracket at C and by a cable BD forming an angle β with the horizontal. (a) Knowing that the ultimate load of the cable is 100 kN, write a computer program to construct a table of the values of the factor of safety of the cable for values of α and β from 0 to 45°, using increments in α and β corresponding to 0.1 increments in tan α and tan β. (b) Check that for any given value of α the maximum value of the factor of safety is obtained for $\beta = 38.66°$ and explain why. (c) Determine the smallest possible value of the factor of safety for $\beta = 38.66°$, as well as the corresponding value of α, and explain the result obtained.

SOLUTION

(a) DRAW F. B. DIAGRAM OF ABC:



$$+\circlearrowleft \Sigma M_C = 0: (P\sin\alpha)(375\text{ mm}) + (P\cos\alpha)(750\text{ mm}) - (F\cos\beta)(375\text{ mm}) - (F\sin\beta)(300\text{ mm}) = 0$$

$$F = P\,\frac{15\sin\alpha + 30\cos\alpha}{15\cos\beta + 12\sin\beta}$$

$$F.S. = F_{ult}/F$$

OUTPUT FOR $P = 16$ kN AND $F_{ult} = 100$ kN. ✓

```
                                  VALUES OF FS
                                       BETA
              0     5.71  11.31  16.70  21.80  26.56  30.96  34.99  38.66  41.99  45.00
 ALPHA
  0.000  3.125  3.358  3.555  3.712  3.830  3.913  3.966  3.994  4.002  3.995  3.977
  5.711  2.991  3.214  3.402  3.552  3.666  3.745  3.796  3.823  3.830  3.824  3.807
 11.310  2.897  3.113  3.295  3.441  3.551  3.628  3.677  3.703  3.710  3.704  3.687
 16.699  2.837  3.049  3.227  3.370  3.477  3.553  3.600  3.626  3.633  3.627  3.611
 21.801  2.805  3.014  3.190  3.331  3.438  3.512  3.560  3.585  3.592  3.586  3.570
 26.565  2.795  3.004  3.179  3.320  3.426  3.500  3.547  3.572  3.579  3.573  3.558
 30.964  2.803  3.013  3.189  3.330  3.436  3.510  3.558  3.583  3.590  3.584  3.568
 34.992  2.826  3.036  3.214  3.356  3.463  3.538  3.586  3.611  3.619  3.612  3.596
 38.660  2.859  3.072  3.252  3.395  3.503  3.579  3.628  3.653  3.661  3.655  3.638
 41.987  2.899  3.116  3.298  3.444  3.554  3.631  3.680  3.706  3.713  3.707  3.690
 45.000  2.946  3.166  3.351  3.499  3.611  3.689  3.739  3.765  3.773  3.767  3.750
```

↑(b)

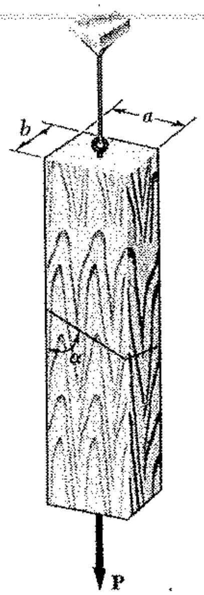
(b) When $\beta = 38.66°$; $\tan\beta = 0.8$ and cable BD is perpendicular to the lever arm BC.

(c) $F.S. = 3.579$ for $\alpha = 26.6°$; P is perpendicular to the lever arm AC

NOTE:

The value $F.S. = 3.579$ is the smallest of the values of F.S. corresponding to $\beta = 38.66°$ and the largest of those corresponding to $\alpha = 26.6°$, The point $\alpha = 26.6°$, $\beta = 38.66°$ is a "saddle point", or "minimax" of the function $F.S.(\alpha, \beta)$.

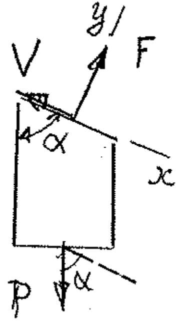
PROBLEM 1.C5



1.C5 A load **P** is supported as shown by two wooden members of uniform rectangular cross section that are joined by a simple glued scarf splice. (*a*) Denoting by σ_U and τ_U, respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of a, b, P, σ_U and τ_U, and for values of α from 5 to 85° at 5° intervals, can be used to calculate (1) the normal stress in the joint, (2) the shearing stress in the joint, (3) the factor of safety relative to failure in tension, (4) the factor of safety relative to failure in shear, (5) the overall factor of safety for the glued joint. (*b*) Apply this program, using the dimensions and loading of the members of Probs. 1.29 and 1.31, knowing that $\sigma_U = 1.26$ MPa and $\tau_U = 1.50$ MPa for the glue used in Prob. 1.29, and that $\sigma_U = 1.03$ MPa and $\tau_U = 1.47$ MPa for the glue used in Prob. 1.31. (*c*) Verify in each of these two cases that the shearing stress is maximum for $\alpha = 45°$.

SOLUTION

(1) and (2) Draw the F.B. diagram of lower member:



$\overset{+}{\searrow}\sum F_x = 0$: $\quad -V + P\cos\alpha = 0 \qquad V = P\cos\alpha$

$+\nearrow\sum F_y = 0$: $\quad F - P\sin\alpha = 0 \qquad F = P\sin\alpha$

Area $= ab/\sin\alpha$

Normal stress:

$$\sigma = \frac{F}{\text{Area}} = (P/ab)\sin^2\alpha$$

Shearing stress: $\tau = \dfrac{V}{\text{Area}} = (P/ab)\sin\alpha\cos\alpha$

(3) F.S. for tension (normal stresses)

$FSN = \sigma_U/\sigma$

(4) F.S. for shear:

$FSS = \tau_U/\tau$

(5) OVERALL F.S.:

FS = The smaller of FSN and FSS.

(CONTINUED)

PROGRAM OUTPUTS

```
Problem 1.31
    a =    150  mm
    b =    75   mm
    P =    11   kN
    SIGU =    1.26 MPa
    TAUU =    1.50 MPa

   ALPHA   SIG (MPa)  TAU (MPa)  FSN        FSS        FS

     5       .007       .085   169.644    17.669    17.669
    10       .029       .167    42.736     8.971     8.971
    15       .065       .244    19.237     6.136     6.136
    20       .114       .314    11.016     4.773     4.773
    25       .175       .375     7.215     4.005     4.005
    30       .244       .423     5.155     3.543     3.543
    35       .322       .459     3.917     3.265     3.265
    40       .404       .481     3.119     3.116     3.116
    45       .489       .489     2.577     3.068     2.577    ◄ (b), (c)
    50       .574       .481     2.196     3.116     2.196
    55       .656       .459     1.920     3.265     1.920
    60       .733       .423     1.718     3.543     1.718
    65       .803       .375     1.569     4.005     1.569
    70       .863       .314     1.459     4.773     1.459
    75       .912       .244     1.381     6.136     1.381
    80       .948       .167     1.329     8.971     1.329
    85       .970       .085     1.298    17.669     1.298
```

```
Problem 1.29
    a =    125 mm
    b =    75 mm
    P =    5600 N
    SIGU =    1.05 MPa
    TAUU =    1.5 MPa
```

ALPHA	SIG (kPa)	TAU (kPa)	FSN	FSS	FS
5	4.693	56.728	211.574	26.408	26.408
25	116.69	250.243	8.998	5.986	5.986
45	326.669	321.769	3.214	4.586	3.214
60	490.00	282.905	2.143	5.295	2.143
85	648.368	55.877	1.619	26.408	1.619

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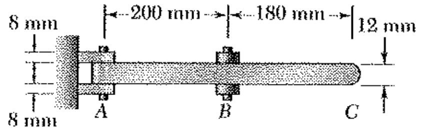
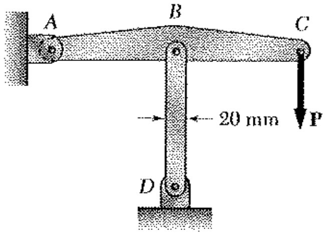
PROBLEM 1.C6

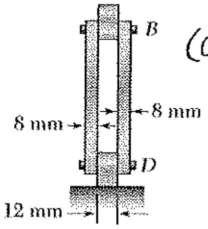
1.C6 Member *ABC* is supported by a pin and bracket at *A* and by two links, which are pin-connected to the member at *B* and to a fixed support at *D*. (*a*) Write a computer program to calculate the allowable load P_{all} for any given values of (1) the diameter d_1 of the pin at *A*, (2) the common diameter d_2 of the pins at *B* and *D*, (3) the ultimate normal stress σ_U in each of the two links, (4) the ultimate shearing stress τ_U in each of the three pins, (5) the desired overall factor of safety *F.S.* Your program should also indicate which of the following three stresses is critical: the normal stress in the links, the shearing stress in the pin at *A*, or the shearing stress in the pins at *B* and *D*. (*b* and *c*) Check your program by using the data of Probs. 1.55 and 1.56, respectively, and comparing the answers obtained for P_{all} with those given in the text. (*d*) Use your program to determine the allowable load P_{all}, as well as which of the stresses is critical, when $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MP for aluminum links, $\tau_U = 100$ MPa for steel pins, and *F.S.* = 3.2.

Top view





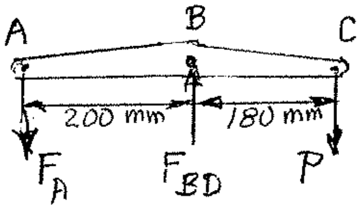
Front view



Side view

SOLUTION

(a) F.B. DIAGRAM OF ABC:



$\sum M_A = 0$: $P = \frac{200}{380} F_{BD}$

$\sum M_B = 0$: $P = \frac{200}{180} F_A$

(1) For given d_1 of pin A: $F_A = 2(\tau_U/FS)(\pi d_1^2/4)$, $P_1 = \frac{200}{180} F_A$

(2) For given d_2 of pins B and D: $F_{BD} = 2(\tau_U/FS)(\pi d_2^2/4)$, $P_2 = \frac{200}{380} F_{BD}$

(3) For ultimate stress in links BD: $F_{BD} = 2(\sigma_U/FS)(0.02)(0.008)$, $P_3 = \frac{200}{380} F_{BD}$

(4) For ult. shearing stress in pins: P_4 is the smaller of P_1 and P_2

(5) For desired overall F.S.: P_5 is the smaller of P_3 and P_4

If $P_3 < P_4$, stress is critical in links
If $P_4 < P_3$ and $P_1 < P_2$, stress is critical in pin A
If $P_4 < P_3$ and $P_2 < P_1$, stress is critical in pins B and D

PROGRAM OUTPUTS

(b) Prob. 1.53. DATA: $d_1 = 8$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, F.S. = 3.0
$P_{all} = 3.72$ kN. Stress in pin A is critical ◀

(c) Prob. 1.54 DATA: $d_1 = 10$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, F.S. = 3.0
$P_{all} = 3.97$ kN., Stress in pins B and D is critical ◀

(d) DATA: $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa, $\tau_U = 100$ MPa, F.S. = 3.2
$P_{all} = 5.79$ kN, Stress in links is critical ◀

Chapter 2

Problem 2.1

2.1 Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod. Knowing that, with an axial load of 6000 N acting on the rod, the distance between the gage marks is 250.18 mm, determine the modulus of elasticity of the aluminum used in the rod.

$\delta = \Delta L = L - L_o = 250.18 - 250.00 = 0.18 \text{ mm}$

$\varepsilon = \frac{\delta}{L_o} = \frac{0.18 \text{ mm}}{250 \text{ mm}} = 0.00072$

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$

$\sigma = \frac{P}{A} = \frac{6000}{113.097 \times 10^{-6}} = 53.052 \times 10^6 \text{ Pa}$

$E = \frac{\sigma}{\varepsilon} = \frac{53.052 \times 10^6}{0.00072} = 73.683 \times 10^9 \text{ Pa}$ $\quad E = 73.7 \text{ GPa}$ ◄

Problem 2.2

2.2 A polystyrene rod of length 300 mm and diameter 12 mm is subjected to a 3-kN tensile load. Knowing that $E = 3.1$ GPa, determine (*a*) the elongation of the rod, (*b*) the normal stress in the rod.

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.1 \times 10^{-6} \text{ m}^2$

(a) $\delta = \frac{PL}{AE} = \frac{(3000)(0.3)}{(113.1 \times 10^{-6})(3.1 \times 10^9)} = 0.002567 \text{ m}$ $\quad \delta = 2.6 \text{ mm}$ ◄

(b) $\sigma = \frac{P}{A} = \frac{3000}{113.1 \times 10^{-6}} = 26.525 \text{ Pa}$ $\quad \sigma = 26.5 \text{ MPa}$ ◄

Problem 2.3

2.3 A 60-m-long steel wire is subjected to 6 kN tensile force. Knowing that E = 200 GPa and that the length of the rod increases by 48 mm, determine (*a*) the smallest diameter that may be selected for the wire, (*b*) the corresponding normal stress.

$P = 6 \times 10^3 \text{ N}, \quad \delta = 48 \times 10^{-3} \text{ m}, \quad E = 200 \times 10^9 \text{ Pa}$ $\quad \delta = \frac{PL}{AE}$

$A = \frac{PL}{E\delta} = \frac{(6 \times 10^3)(60)}{(200 \times 10^9)(48 \times 10^{-3})} = 37.5 \times 10^{-6} \text{ m}^2$

(a) $A = \frac{\pi}{4} d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(37.5 \times 10^{-6})}{\pi}} = 6.91 \times 10^{-3} \text{ m}$ $\quad d = 6.91 \text{ mm}$ ◄

(b) $\sigma = \frac{P}{A} = \frac{6 \times 10^3}{37.5 \times 10^{-6}} = 160 \times 10^6 \text{ Pa}$ $\quad \sigma = 160.0 \text{ MPa}$ ◄

Problem.2.4

2.4 A 9-m length of 6 mm-diameter steel wire is to be used in a hanger. It is noted that the wire stretches 11 mm when a tensile force **P** is applied. Knowing that $E = 200$ GPa, determine (*a*) the magnitude of the force **P**, (*b*) the corresponding normal stress in the wire

$L = 9 \text{ m}$

$A = \frac{\pi}{4} d^2 = (\frac{\pi}{4})(0.006)^2 = 28.27 \times 10^{-6} \text{ m}^2$

(a) $\delta = \frac{PL}{AE} \quad P = \frac{AE\delta}{L} = \frac{(28.27 \times 10^{-6})(200 \times 10^9)(0.011)}{9} = 6.91 \text{ kN}$ ◄

(b) $\sigma = E\varepsilon = \frac{E\delta}{L} = \frac{(200 \times 10^9)(0.011)}{9} = 244.4 \text{ MPa}$ ◄

Problem 2.5

2.5 A cast-iron tube is used to support a compressive load. Knowing that $E = 69$ GPa and that the maximum allowable change in length is 0.025 %, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 7.2 kN if the outside diameter of the tube is 50 mm.

$E = 69 \text{ GPa} = 69 \times 10^9 \text{ Pa}$

$\varepsilon = \frac{\delta}{L} = \frac{0.00025\,L}{L} = 0.00025$

(a) $\sigma = E\varepsilon = (69\times10^9)(0.00025) = 17.25\times10^6 \text{ Pa}$ $\quad \sigma = 17.25 \text{ MPa}$ ◀

(b) $\sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{7.2\times10^3}{17.25\times10^6} = 417.39\times10^{-6}\text{ m}^2 = 417.39 \text{ mm}^2$

$A = \frac{\pi}{4}(d_o^2 - d_i^2) \quad d_i^2 = d_o^2 - \frac{4A}{\pi} = 50^2 - \frac{(4)(417.39)}{\pi} = 1968.56 \text{ mm}^2$

$d_i = 44.368 \text{ mm} \quad t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(50 - 44.368)$

$t = 2.82 \text{ mm}$ ◀

Problem 2.6

2.6 A control rod made of yellow brass must not stretch more than 3 mm when the tension in the wire is 4 kN. Knowing that $E = 105$ GPa and that the maximum allowable normal stress is 180 MPa, determine (*a*) the smallest diameter that can be selected for the rod, (*b*) the corresponding maximum length of the rod.

(a) $\sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{4\times10^3}{180\times10^6} = 22.222\times10^{-6}\text{ m}^2$

$A = \frac{\pi}{4}d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(22.222\times10^{-6})}{\pi}} = 5.32\times10^{-3}\text{ m}$

$d = 5.32 \text{ mm}$ ◀

(b) $\delta = \frac{PL}{AE} \quad L = \frac{AE\delta}{P} = \frac{(22.222\times10^{-6})(105\times10^9)(3\times10^{-3})}{4\times10^3}$

$L = 1.750 \text{ m}$ ◀

Problem 2.7

2.7 Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod with $E = 73$ GPa and an ultimate strength of 140 MPa. Knowing that the distance between the gage marks is 250.28 mm after a load is applied, determine (*a*) the stress in the rod, (*b*) the factor of safety.

$E = 73\times10^9 \text{ Pa}$

$\delta = 250.28 - 250.00 = 0.28 \text{ mm} = 0.28\times10^{-3}\text{ m}$

$L = 250 \text{ mm} = 250\times10^{-3}\text{ m}$

(a) $\sigma = E\varepsilon = \frac{E\delta}{L} = \frac{(73\times10^9)(0.28\times10^{-3})}{250\times10^{-3}} = 81.76\times10^6 \text{ Pa} \quad 81.8 \text{ MPa}$ ◀

(b) $\text{F.S.} = \frac{\sigma_U}{\sigma} = \frac{140\times10^6}{81.76\times10^6} = 1.712$ ◀

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Problem 2.8

2.8 An 80-m-long wire of 5-mm diameter is made of a steel with $E = 200$ GPa and an ultimate tensile strength of 400 MPa. If a factor of safety of 3.2 is desired, determine (*a*) the largest allowable tension in the wire, (*b*) the corresponding elongation of the wire.

(a) $\sigma_U = 400\times10^6$ Pa $\qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(5)^2 = 19.635\ \text{mm}^2 = 19.635\times10^{-6}\ \text{m}^2$

$P_U = \sigma_U A = (400\times10^6)(19.635\times10^{-6}) = 7854$ N

$P_{all} = \frac{P_U}{F.S.} = \frac{7854}{3.2} = 2454$ N $\qquad P_{all} = 2.45$ kN ◀

(b) $\delta = \frac{PL}{AE} = \frac{(2454)(80)}{(19.635\times10^{-6})(200\times10^9)} = 50.0\times10^{-3}$ m

$\delta = 50.0$ mm ◀

Problem 2.9

2.9 A block of 250-mm length and 50 × 40 mm cross section is to support a centric compressive load **P**. The material to be used is a bronze for which $E = 95$ GPa. Determine the largest load which can be applied, knowing that the normal stress must not exceed 80 MPa and that the decrease in length of the block should be at most 0.12% of its original length.

$A = (50)(40) = 2000\ \text{mm}^2$
$= 2\times10^{-3}\ \text{m}^2$

$\sigma_U = 80$ MPa $= 80\times10^6$ Pa $\qquad E = 95\times10^9$ Pa

Considering allowable stress:

$\sigma = \frac{P}{A} \qquad P = A\sigma = (2\times10^{-3})(80\times10^6) = 160\times10^3$ N

Considering allowable deformation:

$\delta = \frac{PL}{AE} \qquad P = AE\left(\frac{\delta}{L}\right) = (2\times10^{-3})(95\times10^9)(0.0012) = 228\times10^3$ N

The smaller value governs. $\quad P = 160\times10^3$ N $\qquad P = 160.0$ kN ◀

Problem 2.10

2.10 A 1.5-m-long aluminum rod must not stretch more than 1 mm and the normal stress must not exceed 40 MPa when the rod is subjected to a 3-kN axial load. Knowing that $E = 70$ GPa, determine the required diameter of the rod.

$L = 1.5$ m

$\delta = 1\times10^{-3}$ m, $\quad \sigma = 40\times10^6$ Pa, $\quad E = 70\times10^9$ Pa, $\quad P = 3\times10^3$ N

Stress: $\sigma = \frac{P}{A} \qquad A = \frac{P}{\sigma} = \frac{3\times10^3}{40\times10^6} = 75\times10^{-6}\ \text{m}^2 = 75\ \text{mm}^2$

Deformation: $\delta = \frac{PL}{AE}$

$A = \frac{PL}{E\delta} = \frac{(3\times10^3)(1.5)}{(70\times10^9)(1\times10^{-3})} = 64.29\times10^{-6}\ \text{m}^2 = 64.29\ \text{mm}^2$

Larger value of A governs. $\qquad A = 75\ \text{mm}^2$

$A = \frac{\pi}{4}d^2 \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(75)}{\pi}}$ $\qquad d = 9.77$ mm ◀

Problem 2.11

2.11 An aluminum control rod must stretch 2 mm when a 2-kN tensile load is applied to it. Knowing that σ_{all} = 154 MPa and E = 70 GPa, determine the smallest diameter and shortest length which may be selected for the rod.

$P = 2\ kN$, $\delta = 2\ mm$ $\quad \sigma_{all} = 154\ MPa$

$\sigma = \frac{P}{A} \leqslant \sigma_{all} \qquad A \geqslant \frac{P}{\sigma_{all}} = \frac{2000}{154} = 12.987\ mm^2$

$A = \frac{\pi}{4} d^2 \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(12.987)}{\pi}} \qquad d_{min} = 4.07\ mm$ ◄

$\sigma = E\varepsilon = \frac{E\delta}{L} \leqslant \sigma_{all}$

$L \geqslant \frac{E\delta}{\sigma_{all}} = \frac{(70 \times 10^9)(0.002)}{154 \times 10^6} = 0.909\ m \qquad L_{min} = 0.91\ m$ ◄

Problem 2.12

2.12 A square aluminum bar should not stretch more than 1.4 mm when it is subjected to a tensile load. Knowing that E = 70 GPa and that the allowable tensile strength is 120 MPa, determine (*a*) the maximum allowable length of the pipe, (*b*) the required dimensions of the cross-section if the tensile load is 28 kN.

$\sigma = 120 \times 10^6\ Pa$

$E = 70 \times 10^9\ Pa \qquad \delta = 1.4 \times 10^{-3}\ m$

(a) $\delta = \frac{PL}{AE} = \frac{\sigma L}{E} \qquad L = \frac{E\delta}{\sigma} = \frac{(70 \times 10^9)(.4 \times 10^{-3})}{120 \times 10^6} = 0.817\ m$

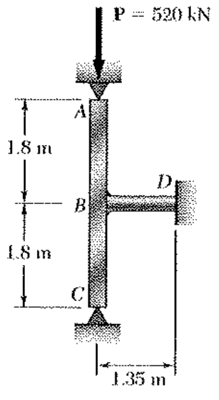
$L = 817\ mm$ ◄

(b) $\sigma = \frac{P}{A} \qquad A = \frac{P}{\sigma} = \frac{28 \times 10^3}{120 \times 10^6} = 233.333 \times 10^{-6}\ m^2 = 233.333\ mm^2$

$A = a^2 \qquad a = \sqrt{A} = \sqrt{233.333} \qquad a = 15.28\ mm$ ◄

Problem 2.13

2.13 Rod BD is made of steel (E = 200 GPa) and is used to brace the axially compressed member ABC. The maximum force that can be developed in member BD is 0.02P. If the stress must not exceed 126 MPa and the maximum change in length of BD must not exceed 0.001 times the length of ABC, determine the smallest-diameter rod that can be used for member BD.



$F_{BD} = 0.02\ P = (0.02)(520) = 10.4\ kN$

Considering stress: $\sigma = 126\ MPa$

$\sigma = \frac{F_{BD}}{A} \therefore A = \frac{F_{BD}}{\sigma} = \frac{10400}{126} = 82.54\ mm^2$

Considering deformation: $\delta = (0.001)(3600) = 3.6\ mm$

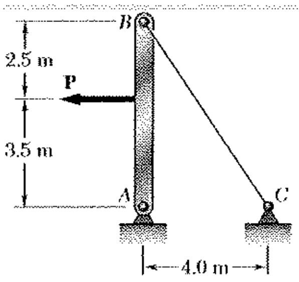
$\delta = \frac{F_{BD} L_{BD}}{AE} \therefore A = \frac{F_{BD} L_{BD}}{E\delta} = \frac{(10.4 \times 10^3)(1350)}{(200 \times 10^3)(3.6)} = 19.5\ mm^2$

Larger area governs. $A = 19.5\ mm^2$

$A = \frac{\pi}{4} d^2 \therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(19.5)}{\pi}} \qquad d = 5\ mm$ ◄

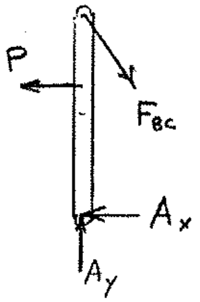
Problem 2.14

2.14 The 4-mm-diameter cable BC is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.



$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111$ m

Use bar AB as a free body.



$\circlearrowleft \Sigma M_A = 0 \quad 3.5P - (6)(\frac{4}{7.2111} F_{BC}) = 0$

$P = 0.9509 \, F_{BC}$

Considering allowable stress: $\sigma = 190 \times 10^6$ Pa

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6}$ m^2

$\sigma = \frac{F_{BC}}{A} \therefore F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3$ N

Considering allowable elongation: $\delta = 6 \times 10^{-3}$ m

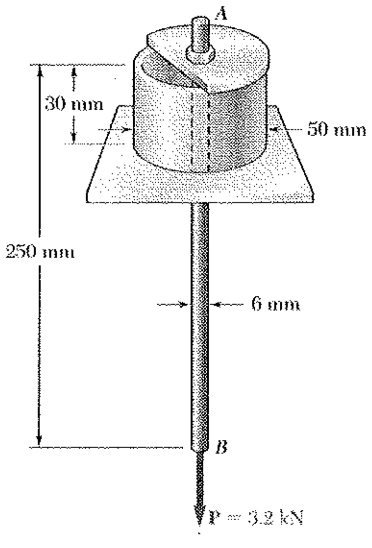
$\delta = \frac{F_{BC} L_{BC}}{AE} \therefore F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3$ N

Smaller value governs. $F_{BC} = 2.091 \times 10^3$ N

$P = 0.9509 \, F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3$ N $\qquad P = 1.988$ kN ◄

Problem 2.15

2.15 A 3-mm-thick hollow polystyrene cylinder ($E = 3$ GPa) and a rigid circular plate (only part of which is shown) are used to support a 250-mm-long steel rod AB ($E = 200$ GPa) of 6 mm diameter. If an 3.2 kN load **P** is applied at B, determine (*a*) the elongation of rod AB, (*b*) the deflection of point B, (*c*) the average normal stress in rod AB.



Rod AB: $P_{AB} = 3200$ N $\quad L_{AB} = 0.25$ m $\quad d = 6$ mm

$A_{AB} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.006)^2 = 28.27 \times 10^{-6}$ m^2

(a) $\delta_{AB} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} = \frac{(3200)(0.25)}{(200 \times 10^9)(28.27 \times 10^{-6})} = 0.1415 \times 10^{-3}$ m ◄

Hollow cylinder: $d_o = 0.05$ m $\quad d_i = (0.05 - (0.003)(2)) = 0.044$ m

$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(0.05^2 - 0.044^2) = 0.000443$ m^2

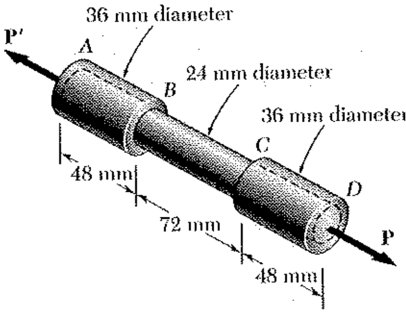
$L = 0.03$ m, $\quad P = 3200$ N

$\delta_{cyl} = \frac{PL}{EA} = \frac{(3200)(0.03)}{(3 \times 10^9)(0.000443)} = 0.0722 \times 10^{-3}$ m

(b) Deflection of point B $\quad \delta_B = \delta_{AB} + \delta_{cyl} = \quad 0.214 \times 10^{-3}$ m $\downarrow$ ◄

(c) Stress in rod AB $\quad \sigma = \frac{P}{A_{AB}} = \frac{3200}{28.27 \times 10^{-6}} = 113.2$ MPa ◄

Problem 2.16



2.16 The specimen shown is made from a 24-mm-diameter cylindrical steel rod with two 36-mm-outer-diameter sleeves bonded to the rod as shown. Knowing that $E = 200$ GPa, determine (*a*) the load **P** so that the total deformation is 0.04 mm, (*b*) the corresponding deformation of the central portion *BC*.

(a) $$\delta = \sum \frac{P_i L_i}{A_i E_i} = \frac{P}{E} \sum \frac{L_i}{A_i}$$

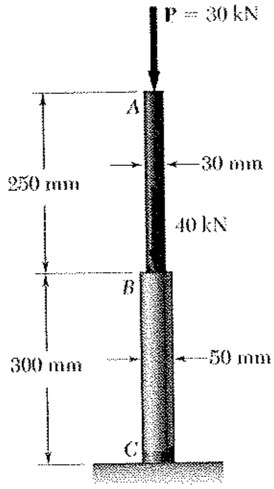
$$P = E\delta \left(\sum \frac{L_i}{A_i}\right)^{-1} \qquad A_i = \frac{\pi}{4} d_i^2$$

	L, mm	d, mm	A, mm²	L/A, mm⁻¹
AB	48	36	1017·9	0·04716
BC	72	24	452·4	0·15915
CD	48	36	1017·9	0·04716
				0·25347 ← sum

$$P = (200\times10^3)(0.04)(0.25347)^{-1} = 31.562\times10^3\ N \qquad P = 31.6\ kN$$ ◄

(b) $$\delta_{BC} = \frac{PL_{BC}}{A_{BC}E} = \frac{P}{E}\frac{L_{BC}}{A_{BC}} = \frac{31.562\times10^3}{200\times10^3}(0.15915) \qquad \delta = 0.025\ mm$$ ◄

Problem 2.17



2.17 Two solid cylindrical rods are joined at *B* and loaded as shown. Rod *AB* is made of steel ($E = 200$ GPa) and rod *BC* of brass ($E = 105$ GPa). Determine (*a*) the total deformation of the composite rod *ABC*, (*b*) the deflection of point *B*.

Rod AB: $F_{AB} = -P = -30\times10^3\ N$

$L_{AB} = 0.250\ m \qquad E_{AB} = 200\times10^9\ GPa$

$A_{AB} = \frac{\pi}{4}(30)^2 = 706.85\ mm^2 = 706.85\times10^{-6}\ m^2$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{E_{AB}A_{AB}} = -\frac{(30\times10^3)(0.250)}{(200\times10^9)(706.85\times10^{-6})}$$

$$= -53.052\times10^{-6}\ m$$

Rod BC: $F_{BC} = 30 + 40 = 70\ kN = 70\times10^3\ N$

$L_{BC} = 0.300\ m \qquad E_{BC} = 105\times10^9\ Pa$

$A_{BC} = \frac{\pi}{4}(50)^2 = 1.9635\times10^3\ mm^2 = 1.9635\times10^{-3}\ m^2$

$$\delta_{BC} = \frac{F_{BC}L_{BC}}{E_{BC}A_{BC}} = -\frac{(70\times10^3)(0.300)}{(105\times10^9)(1.9635\times10^{-3})}$$

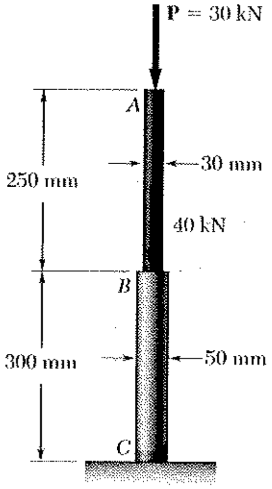
$$= -101.859\times10^{-6}\ m$$

(a) Total deformation: $\delta_{tot} = \delta_{AB} + \delta_{BC} = -154.9\times10^{-6}\ m = -0.1549\ mm$ ◄

(b) Deflection of point B. $\delta_B = \delta_{BC} \qquad \delta_B = 0.1019\ mm \downarrow$ ◄

Problem 2.18

2.18 For the composite rod of Prob. 2.17, determine (*a*) the load *P* for which the total deformation of the rod is -0.2 mm, (*b*) the corresponding deflection of point *B*.



2.17 Two solid cylindrical rods are joined at *B* and loaded as shown. Rod *AB* is made of steel (*E* = 200 GPa) and rod *BC* of brass (*E* = 105 GPa). Determine (*a*) the total deformation of the composite rod *ABC*, (*b*) the deflection of point *B*.

Rod AB: $F_{AB} = -P$

$L_{AB} = 0.250 \text{m} \qquad E_{AB} = 200\times10^9 \text{ Pa}$

$A_{AB} = \frac{\pi}{4}(30)^2 = 706.85 \text{ mm}^2 = 706.85\times10^{-6} \text{ m}^2$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E_{AB} A_{AB}} = -\frac{P\,(0.250)}{(200\times10^9)(706.85\times10^{-6})}$$

$$= -1.76841\ P$$

Rod BC: $F_{BC} = -(P + 40\times10^3)$

$L_{BC} = 0.300 \text{ m} \qquad E_{BC} = 105\times10^9 \text{ Pa}$

$A_{BC} = \frac{\pi}{4}(50)^2 = 1.9635\times10^3 \text{ mm}^2 = 1.9635\times10^{-3} \text{ m}^2$

$$\delta_{BC} = \frac{F_{BC} L_{BC}}{E_{BC} A_{BC}} = -\frac{(P+40\times10^3)(0.300)}{(105\times10^9)(1.9635\times10^{-3})}$$

$$= -1.45513\times10^{-9} P - 58.205\times10^{-6} \qquad (1)$$

(a) Total deformation: $\delta_{tot} = \delta_{AB} + \delta_{BC}$

$$-0.2\times10^{-3} = -1.76841\times10^{-9} P - 1.45513\times10^{-9} P - 58.205\times10^{-6}$$

$P = 43.987\times10^3 \text{ N} \qquad P = 44.0 \text{ kN}$ ◄

From (1), $\delta_{BC} = -(1.45513\times10^{-9})(43.987\times10^3) - 58.205\times10^{-3}$

$$= -122.21\times10^{-6} \text{ m}$$

(b) Deflection of point B. $\delta_B = \delta_{BC} \qquad \delta_B = 0.1222 \text{ mm} \downarrow$ ◄

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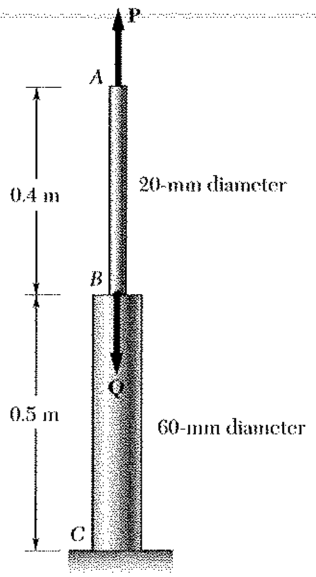
Problem 2.19

2.19 Both portions of the rod *ABC* are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.



(a) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\ m^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\ m^2$

Force in member AB is P tension.

Elongation. $\delta_{AB} = \frac{P L_{AB}}{E A_{AB}} = \frac{(4\times10^3)(0.4)}{(70\times10^9)(314.16\times10^{-6})}$

$= 72.756 \times 10^{-6}\ m$

Force in member BC is Q - P compression.

Shortening. $\delta_{BC} = \frac{(Q-P) L_{BC}}{E A_{BC}} = \frac{(Q-P)(0.5)}{(70\times10^9)(2.8274\times10^{-3})}$

$= 2.5263 \times 10^{-9}\ (Q-P)$

For zero deflection at A, $\delta_{BC} = \delta_{AB}$

$2.5263\times10^{-9}(Q-P) = 72.756\times10^{-6} \quad \therefore \quad Q - P = 28.8\times10^3\ N$

$Q = 28.3\times10^3 + 4\times10^3 = 32.8\times10^3\ N = 32.8\ kN$ ◄

(b) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756\times10^{-6}\ m = 0.0728\ mm \downarrow$ ◄

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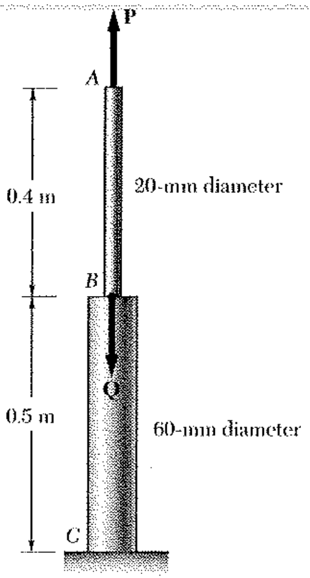
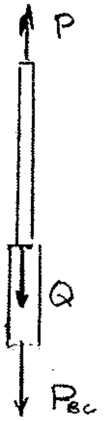
Problem 2.20

2.20 The rod *ABC* is made of an aluminum for which $E = 70$ GPa. Knowing that $P = 6$ kN and $Q = 42$ kN, determine the deflection of (*a*) point *A*, (*b*) point *B*.



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3} \text{ m}^2$$



$$P_{AB} = P = 6 \times 10^3 \text{ N}$$

$$P_{BC} = P - Q = 6 \times 10^3 - 42 \times 10^3 = -36 \times 10^3 \text{ N}$$

$$L_{AB} = 0.4 \text{ m} \qquad L_{BC} = 0.5 \text{ m}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E_A} = \frac{(6 \times 10^3)(0.4)}{(314.16 \times 10^{-6})(70 \times 10^9)}$$

$$= 109.135 \times 10^{-6} \text{ m}$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36 \times 10^3)(0.5)}{(2.8274 \times 10^{-3})(70 \times 10^9)}$$

$$= -90.947 \times 10^{-6} \text{ m}$$

(a) $\delta_A = \delta_{AB} + \delta_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6} \text{ m} = 18.19 \times 10^{-6} \text{ m}$

$= 0.01819 \text{ mm} \uparrow$ ◀

(b) $\delta_B = \delta_{BC} = -90.9 \times 10^{-6} \text{ m} = -0.0909 \text{ mm}$ or $0.0919 \text{ mm} \downarrow$ ◀

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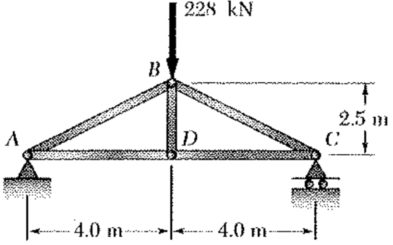
Problem 2.21

2.21 For the steel truss (E = 200 GPa) and loading shown, determine the deformations of the members AB and AD, knowing that their cross-sectional areas are 2400 mm^2 and 1800 mm^2, respectively.



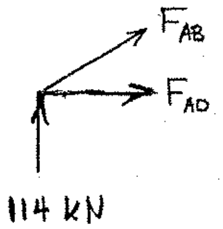
Statics: Reactions are 114 kN upward at A & C.

Member BD is a zero force member.

$L_{AB} = \sqrt{4.0^2 + 2.5^2} = 4.717$ m

Use joint A as a free body. $+\!\uparrow \Sigma F_y = 0$: $114 + \frac{2.5}{4.717} F_{AB} = 0$

$F_{AB} = -215.10$ kN



$\overset{+}{\rightarrow} \Sigma F_x = 0$: $F_{AD} + \frac{4}{4.717} F_{AB} = 0$

$F_{AD} = -\frac{(4)(-215.10)}{4.717} = 182.4$ kN

Member AB: $\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(-215.10\times10^3)(4.717)}{(200\times10^9)(2400\times10^{-6})}$

$= -2.11\times10^{-3}$ m $\qquad \delta_{AB}$ - 2.11 mm ◀

$\delta_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}} = \frac{(182.4\times10^3)(4.0)}{(200\times10^9)(1800\times10^{-6})} = 2.03\times10^{-3}$ m $\qquad \delta_{AC} = 2.03$ mm ◀

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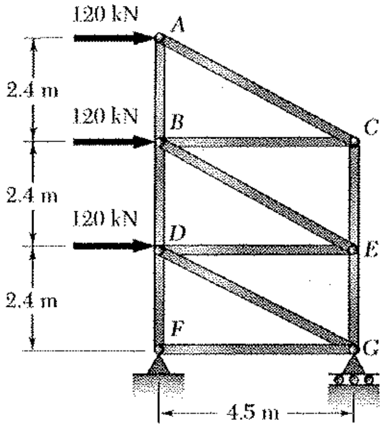
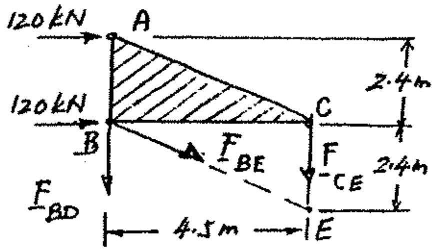
Problem 2.22

2.22 For the steel truss ($E = 200$ GPa) and loading shown, determine the deformations of the members BD and DE, knowing that their cross-sectional areas are 1250 mm^2 and 1875 mm^2, respectively.



Free Body: Portion ABC of truss

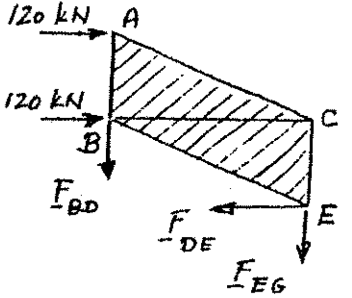


$+\circlearrowleft \Sigma M_E = 0:$

$F_{BD}(4.5\text{ m}) - (120\text{ kN})(2.4\text{ m}) - (120\text{ kN})(4.8\text{ m}) = 0$

$F_{BD} = +192$ kN

Free Body: Portion ABEC of truss



$\xrightarrow{+} \Sigma F_x = 0:$

$120\text{ kN} + 120\text{ kN} - F_{DE} = 0$

$F_{DE} = +240$ kN

$$\delta_{BD} = \frac{PL}{AE} = \frac{(+192\times10^3\text{ N})(2400\text{ mm})}{(1250\text{ mm}^2)(200000\text{ MPa})}$$

$\delta_{BD} = +1.84$ mm ◀

$$\delta_{DE} = \frac{PL}{AE} = \frac{(+240\times10^3\text{ N})(4500\text{ mm})}{(1875\text{ mm}^2)(200000\text{ MPa})}$$

$\delta_{DE} = +2.88$ mm ◀

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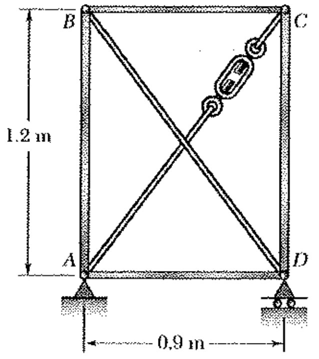
Problem 2.23



2.23 Members *AB* and *CD* are 30-mm-diameter steel rods, and members *BC* and *AD* are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member *AC* is put in tension. Knowing that $E = 200$ GPa, determine the largest allowable tension in *AC* so that the deformations in members *AB* and *CD* do not exceed 1.0 mm.

$\delta_{AB} = \delta_{CD} = 1mm$ $\qquad h = 1.2\,m \qquad = L_{CD}$

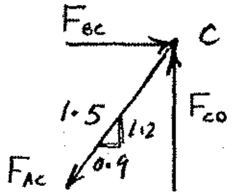
$A_{CD} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.03)^2 = 706.9\times10^{-6}\,m^2$

$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA_{CD}}$

$F_{CD} = \frac{EA_{CD}\delta_{CD}}{L_{CD}} = \frac{(200\times10^9)(706.9\times10^{-6})(0.001)}{1.2}$

$= 117.8\ kN$

Use joint C as a free body



$+\uparrow \Sigma F_y = 0: \quad F_{CD} - \frac{1.2}{1.5}F_{AC} = 0 \quad \therefore F_{AC} = \frac{1.5}{1.2}F_{CD}$

$F_{AC} = \frac{1.5}{1.2}(117.8) = 147.3\,kN.$

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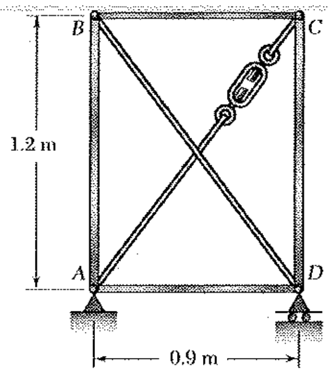
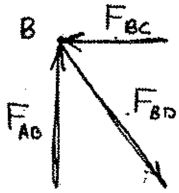
Problem 2.24

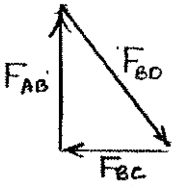
2.24 For the structure in Prob. 2.23, determine (*a*) the distance *h* so that the deformations in members *AB*, *BC*, *CD*, and *AD* are equal to 1 mm, (*b*) the corresponding tension in member *AC*.

2.23 Members *AB* and *CD* are 30-mm-diameter steel rods, and members *BC* and *AD* are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member *AC* is put in tension. Knowing that $E = 200$ GPa and $h = 1.2$ m, determine the largest allowable tension in *AC* so that the deformations in members *AB* and *CD* do not exceed 1 mm.

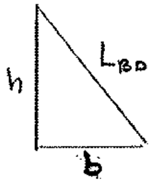


(a) Statics: Use joint B as a free body





Force Triangle



Geometry

From similar triangles

$$\frac{F_{AB}}{h} = \frac{F_{BC}}{b} = \frac{F_{BD}}{L_{BC}}$$

$$F_{AB} = \frac{h}{b} F_{BC}$$

For equal deformations

$$\delta_{AB} = \delta_{BC} \therefore \quad \frac{F_{AB}\, h}{E A_{AB}} = \frac{F_{BC}\, b}{E A_{BC}} \quad \therefore \quad F_{AB} = \frac{b}{h} \cdot \frac{A_{AB}}{A_{BC}} F_{BC}$$

Equating expressions for F_{AB}

$$\frac{h}{b} F_{BC} = \frac{b}{h} \frac{A_{AB}}{A_{BC}} F_{BC} \qquad \frac{h^2}{b^2} = \frac{A_{AB}}{A_{BC}} = \frac{\frac{\pi}{4} d_{AB}^2}{\frac{\pi}{4} d_{BC}^2} = \frac{d_{AB}^2}{d_{BC}^2}$$

$$\frac{h}{b} = \frac{d_{AB}}{d_{BC}} = \frac{30}{22} = \frac{15}{11} \qquad b = 0.9\,m$$

$$h = \frac{15}{11} b = \frac{15}{11}(0.9) = 1.227\,m$$ ◄

(b) Setting $\delta_{AB} = \delta_{BC} = 1\,mm$

$$\delta_{BC} = \frac{F_{BC}\, b}{E A_{BC}} \therefore \qquad F_{BC} = \frac{E A_{BC} \delta_{BC}}{b} = \frac{(200 \times 10^9) \frac{\pi}{4} (0.022)^2 (0.001)}{0.9}$$

$$= 84.5\,kN$$

$$F_{AB} = \frac{h}{b} F_{BC} = \frac{15}{11}(84.5) = 115.2\,kN$$

From the force triangle

$$F_{BD} = F_{AC} = \sqrt{F_{BC}^2 + F_{AB}^2} = 142.9\,kN$$ ◄

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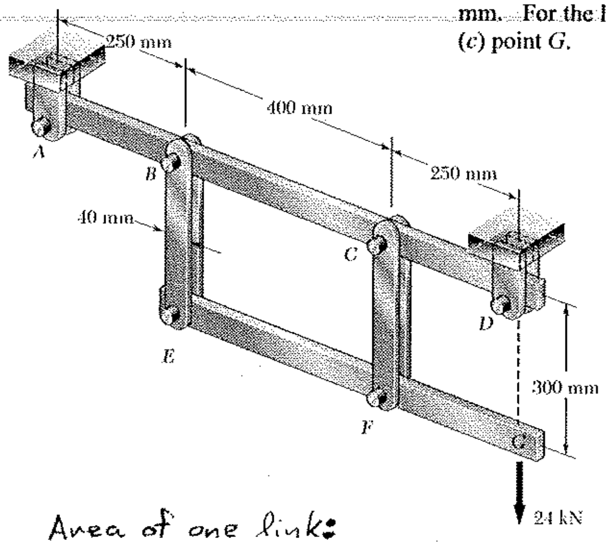
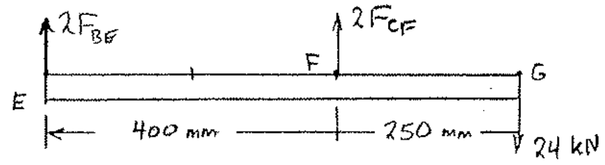
Problem 2.25

2.25 Each of the four vertical links connecting the two horizontal members is made of aluminum ($E = 70$ GPa) and has a uniform rectangular cross section of 10 × 40 mm. For the loading shown, determine the deflection of (*a*) point *E*, (*b*) point *F*, (*c*) point *G*.



Statics. Free body EFG.



$+\circlearrowleft \Sigma M_F = 0:$

$-(400)(2F_{BE}) - (250)(24) = 0$

$F_{BE} = -7.5 \text{ kN} = -7.5\times10^3 \text{ N}$

$+\circlearrowleft \Sigma M_E = 0:$

$(400)(2F_{CF}) - (650)(24) = 0$

$F_{CF} = 19.5 \text{ kN} = 19.5\times10^3 \text{ N}$

Area of one link:

$A = (10)(40) = 400 \text{ mm}^2$

$= 400\times10^{-6} \text{ m}^2$

Length: $L = 300 \text{ mm} = 0.300 \text{ m}$

Deformations.

$$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-7.5\times10^3)(0.300)}{(70\times10^9)(400\times10^{-6})} = -80.357\times10^{-6} \text{ m}$$

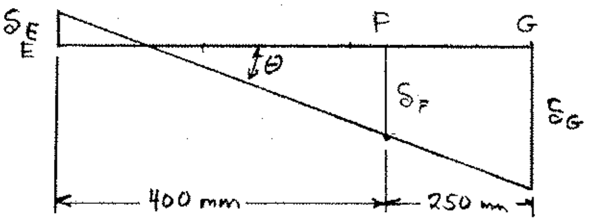
$$\delta_{CF} = \frac{F_{CF}L}{EA} = \frac{(19.5\times10^3)(0.300)}{(70\times10^9)(400\times10^{-6})} = 208.93\times10^{-6} \text{ m}$$

(a) Deflection of point E. $\delta_E = |\delta_{BE}|$ $\delta_E = 80.4\ \mu\text{m} \uparrow$ ◄

(b) Deflection of point F. $\delta_F = \delta_{CF}$ $\delta_F = 209\ \mu\text{m} \downarrow$ ◄

Geometry change.

Let θ be the small change in slope angle.



$$\theta = \frac{\delta_E + \delta_F}{L_{EF}} = \frac{80.357\times10^{-6} + 208.93\times10^{-6}}{0.400} = 723.22\times10^{-6} \text{ radians}$$

(c) Deflection of point G. $\delta_G = \delta_F + L_{FG}\theta$

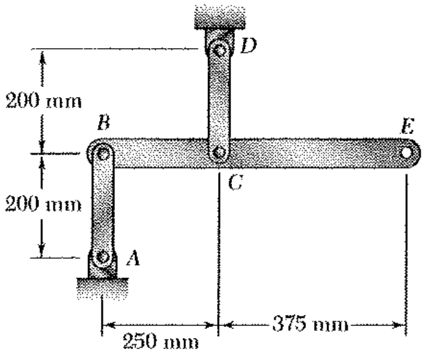
$\delta_G = \delta_F + L_{FG}\theta = 208.93\times10^{-6} + (0.250)(723.22\times10^{-6})$

$= 389.73\times10^{-6}$ m

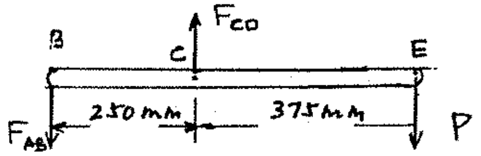
$\delta_G = 390\ \mu\text{m} \downarrow$ ◄

Problem 2.26

2.26 Each of the links *AB* and *CD* is made of steel (E = 200 GPa) and has a uniform rectangular cross section of 6 × 24 mm. Determine the largest load which can be suspended from point *E* if the deflection of *E* is not to exceed 0.25 mm.



Statics. Free body BCE.



$$+\circlearrowleft \Sigma M_C = 0: \quad 0.25 F_{AB} - 0.375 P = 0$$

$$F_{AB} = 1.5 P$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.25 F_{CD} - 0.625 P = 0$$

$$F_{CD} = 2.5 P$$

Area of link:

$A = (6)(24) = 144 \text{ mm}^2$

Length: 200 mm

$L =$

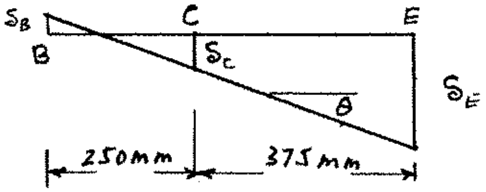
Deformations. $\delta_{AB} = \dfrac{F_{AB} L}{EA} = \dfrac{(1.5P)(0.2)}{(200\times10^9)(144\times10^{-6})} = 1.0417 \times 10^{-8} P$

$\delta_{CD} = \dfrac{F_{CD} L}{EA} = \dfrac{(2.5P)(0.2)}{(200\times10^9)(144\times10^{-6})} = 1.7361 \times 10^{-8} P$

Deflections. Point B. $\delta_B = \delta_{AB}$ $\quad \delta_B = 1.0417 \times 10^{-8} P \uparrow$

Point C. $\delta_C = \delta_{CD}$ $\quad \delta_C = 1.7361 \times 10^{-8} P \downarrow$

Geometry change.



$$\theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{1.0417\times10^{-8}P + 1.7361\times10^{-8}P}{0.25} = 0.11111\times10^{-6} P$$

$$\delta_E = \delta_F + L_{CE}\theta = 1.7361\times10^{-8}P + (0.375)(0.11111\times10^{-6}P) = 5.9027\times10^{-8}P \downarrow$$

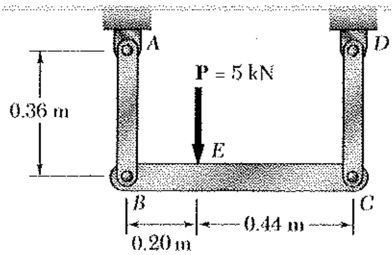
Limiting value of δ_E: $\quad \delta_E = 0.25\times10^{-3}$ m

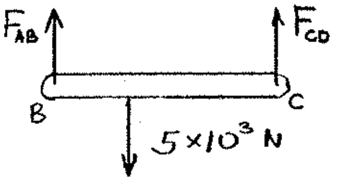
Limiting value of P: $\quad 5.9027 \times 10^{-8} P = 0.25\times10^{-3}$

$P = 4235$ N $\qquad\qquad P = 4.24$ kN ◀

Problem 2.27

2.27 Each of the links AB and CD is made of aluminum ($E = 75$ GPa) and has a cross-sectional area of 125 mm^2. Knowing that they support the rigid member BC, determine the deflection of point E.





Use member BC as a free body.

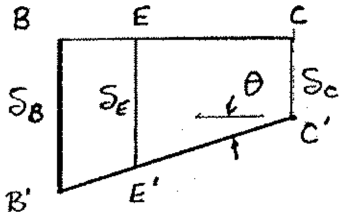
$\circlearrowleft + \; \Sigma M_C = 0: \quad -(0.64)F_{AB} + (0.44)(5\times 10^3) = 0 \qquad F_{AB} = 3.4375\times 10^3$ N

$\circlearrowleft + \; \Sigma M_B = 0: \quad (0.64)F_{CD} - (0.20)(5\times 10^3) = 0 \qquad F_{CD} = 1.5625\times 10^3$ N

For links AB and CD, $\quad A = 125 \text{ mm}^2 = 125\times 10^{-6} \text{ m}^2$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{EA} = \frac{(3.4375\times 10^3)(0.36)}{(75\times 10^9)(125\times 10^{-6})} = 132.00\times 10^{-6} \text{ m} = \delta_B$$

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA} = \frac{(1.5625\times 10^3)(0.36)}{(75\times 10^9)(125\times 10^{-6})} = 60.00\times 10^{-6} \text{ m} = \delta_C$$



Deformation diagram.

$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{l_{BC}} = \frac{72.00\times 10^{-6}}{0.64}$$

$$= 112.5\times 10^{-6} \text{ rad}$$

$$\delta_E = \delta_C + l_{EC}\,\theta$$

$$= 60.00\times 10^{-6} + (0.44)(112.5\times 10^{-6})$$

$$= 109.5\times 10^{-6} \text{ m} \qquad \delta_E = 0.1095 \text{ mm} \downarrow \blacktriangleleft$$

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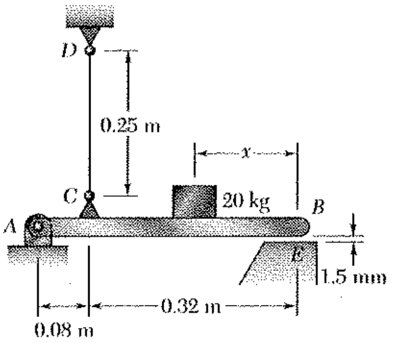
Problem 2.28

2.28 The length of the 2-mm-diameter steel wire *CD* has been adjusted so that with no load applied, a gap of 1.5 mm exists between the end *B* of the rigid beam *ACB* and a contact point *E*. Knowing that *E* = 200 GPa, determine where a 20-kg block should be placed on the beam in order to cause contact between *B* and *E*.



Rigid rod ACE rotates through angle θ to close gap.

$$\theta = \frac{1.5\times10^{-3}}{0.40} = 3.75\times10^{-3} \text{ rad}$$

Point C moves downward.

$$\delta_C = 0.08\,\theta = (0.08)(3.75\times10^{-3}) = 300\times10^{-6} \text{ m}$$

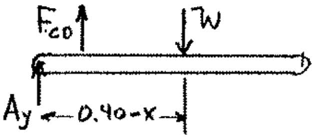
$$\delta_{CD} = \delta_C = 300\times10^{-6} \text{ m}$$

$$A_{CD} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(2)^2 = 3.1416 \text{ mm}^2 = 3.1416\times10^{-6} \text{ m}^2$$

$E = 200\times10^9$ Pa $\qquad \delta_{CD} = \frac{F_{CD}L_{CD}}{EA_{CD}}$

$$F_{CD} = \frac{EA_{CD}\delta_{CD}}{L_{CD}} = \frac{(200\times10^9)(3.1416\times10^{-6})(300\times10^{-6})}{0.25} = 753.98 \text{ N}$$

Use beam ACB as a free body. $\quad W = mg = (20)(9.81) = 196.2$ N



$$+\circlearrowleft \Sigma M_A = 0: \quad 0.08\,F_{CD} - (0.40 - x)W = 0$$

$$0.40 - x = \frac{(0.08)(753.98)}{196.2} = 0.30743 \text{ m}$$

$$x = 0.0926 \text{ m} = 92.6 \text{ mm}$$

For contact, $x < 92.6$ mm ◄

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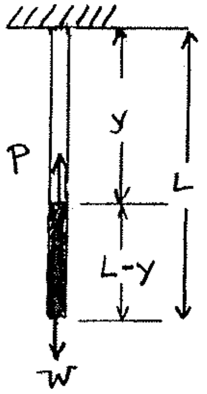
Problem 2.29

2.29 A homogeneous cable of length L and uniform cross section is suspended from one end. (*a*) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (*b*) Show that the same elongation would be obtained if the cable were horizontal and if a force equal to half of its weight were applied at each end.



(a) For element at point identified by coordinate y

$$P = \text{weight of portion below the point}$$

$$= \rho g A (L-y)$$

$$d\delta = \frac{P\,dy}{EA} = \frac{\rho g A (L-y) dy}{EA} = \frac{\rho g (L-y)}{E}\,dy$$

$$\delta = \int_0^L \frac{\rho g (L-y)}{E}\,dy = \frac{\rho g}{E}\left(Ly - \tfrac{1}{2}y^2\right)\Big|_0^L$$

$$= \frac{\rho g}{E}\left(L^2 - \frac{L^2}{2}\right) \qquad \delta = \frac{1}{2}\frac{\rho g L^2}{E} \blacktriangleleft$$

(b) Total weight. $W = \rho g A L$

$$F = \frac{EA\delta}{L} = \frac{EA}{L}\cdot\frac{1}{2}\frac{\rho g L^2}{E} = \tfrac{1}{2}\rho g A L \qquad F = \tfrac{1}{2}W \blacktriangleleft$$

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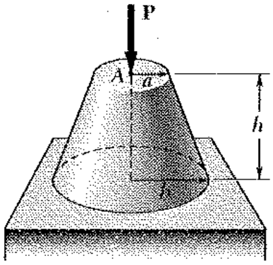
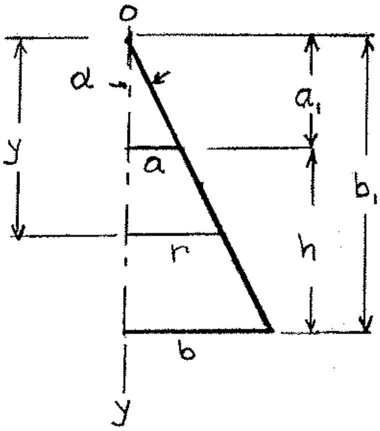
Problem 2.30



2.30 A vertical load **P** is applied at the center *A* of the upper section of a homogeneous frustum of a circular cone of height *h*, minimum radius *a*, and maximum radius *b*. Denoting by *E* the modulus of elasticity of the material and neglecting the effect of its weight, determine the deflection of point *A*.

Extend the slant sides of the cone to meet at a point O and place the origin of the coordinate system there.



From geometry, $\tan\alpha = \frac{b-a}{h}$

$$a_1 = \frac{a}{\tan\alpha}, \quad b_1 = \frac{b}{\tan\alpha}, \quad r = y\tan\alpha$$

At coordinate point y, $A = \pi r^2$

Deformation of element of height dy: $d\delta = \frac{P\,dy}{AE}$

$$d\delta = \frac{P}{E\pi}\frac{dy}{r^2} = \frac{P}{\pi E\tan^2\alpha}\frac{dy}{y^2}$$

Total deformation.

$$\delta_A = \frac{P}{\pi E\tan^2\alpha}\int_{a_1}^{b_1}\frac{dy}{y^2} = \frac{P}{\pi E\tan^2\alpha}\left(-\frac{1}{y}\right)\Bigg|_{a_1}^{b_1} = \frac{P}{\pi E\tan^2\alpha}\left(\frac{1}{a_1}-\frac{1}{b_1}\right)$$

$$= \frac{P}{\pi E\tan^2\alpha}\,\frac{b_1-a_1}{a_1 b_1} = \frac{P(b_1-a_1)}{\pi E ab} \qquad \delta_A = \frac{Ph}{\pi Eab}\downarrow \blacktriangleleft$$

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Problem 2.31

2.31 Denoting by ϵ the "engineering strain" in a tensile specimen, show that the true strain is $\epsilon_t = \ln(1 + \epsilon)$.

$$\varepsilon_t = \ln\frac{L}{L_o} = \ln\frac{L_o + \delta}{L_o} = \ln\left(1 + \frac{\delta}{L_o}\right) = \ln(1+\varepsilon)$$

Thus $\varepsilon_t = \ln(1+\varepsilon)$ ◄

Problem 2.32

2.32 The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is d_1, show that when the diameter is d, the true strain is $\epsilon_t = 2\ln(d_1/d)$.

If the volume is constant, $\frac{\pi}{4}d^2L = \frac{\pi}{4}d_1^2L_o$

$$\frac{L}{L_o} = \frac{d_1^2}{d^2} = \left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = \ln\frac{L}{L_o} = \ln\left(\frac{d_1}{d}\right)^2$$

$\varepsilon_t = 2\ln\frac{d_1}{d}$ ◄

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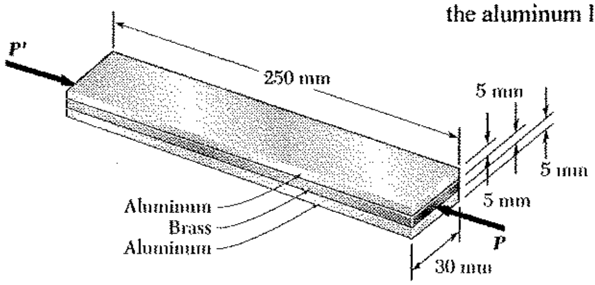
Problem 2.33

2.33 A 250-mm bar of 15 × 30-mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude $P = 30$ kN, and knowing that $E_a = 70$ GPa and $E_b = 105$ GPa, determine the normal stress (*a*) in the aluminum layers, (*b*) in the brass layer.



For each layer,

$A = (30)(5) = 150 \text{ mm}^2 = 150 \times 10^{-6} \text{ m}^2$

Let P_a = load on each aluminum layer.

P_b = load on brass layer.

Deformation. $\delta = \dfrac{P_a L}{E_a A} = \dfrac{P_b L}{E_b A}$ $\qquad P_b = \dfrac{E_b}{E_a} P_a = \dfrac{105}{70} P_a = 1.5 P_a$

Total force. $P = 2P_a + P_b = 3.5 P_a$

Solving for P_a and P_b, $\quad P_a = \frac{2}{7} P \qquad P_b = \frac{3}{7} P$

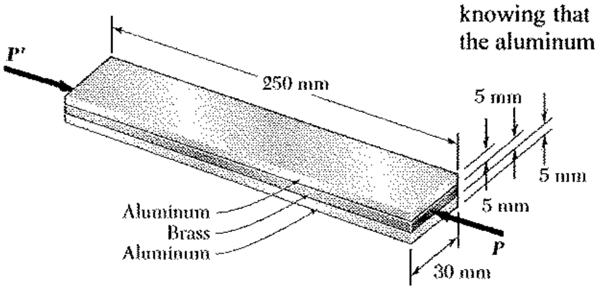
(a) $\sigma_a = -\dfrac{P_a}{A} = -\dfrac{2}{7}\dfrac{P}{A} = -\dfrac{2}{7}\dfrac{30 \times 10^3}{150 \times 10^{-6}} = -57.1 \times 10^6 \text{ Pa}$ $\qquad \sigma_a = -57.1 \text{ MPa}$ ◄

(b) $\sigma_b = -\dfrac{P_b}{A} = -\dfrac{3}{7}\dfrac{P}{A} = -\dfrac{3}{7}\dfrac{30 \times 10^3}{150 \times 10^{-6}} = -85.7 \times 10^6 \text{ Pa}$ $\qquad \sigma_b = -85.7 \text{ MPa}$ ◄

Problem 2.34

2.34 Determine the deformation of the composite bar of Prob. 2.33 if it is subjected to centric forces of magnitude $P = 45$ kN.

2.33 A 250-mm bar of 15 × 30-mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude $P = 30$ kN, and knowing that $E_a = 70$ GPa and $E_b = 105$ GPa, determine the normal stress (*a*) in the aluminum layers, (*b*) in the brass layer.



For each layer,

$A = (30)(5) = 150 \text{ mm}^2 = 150 \times 10^{-6} \text{ m}^2$

Let P_a = load on each aluminum layer.

P_b = load on brass layer.

Deformation. $\delta = -\dfrac{P_a L}{E_a A} = -\dfrac{P_b L}{E_b A}$ $\qquad P_b = \dfrac{E_b}{E_a} P_a = \dfrac{105}{70} P_a = 1.5 P_a$

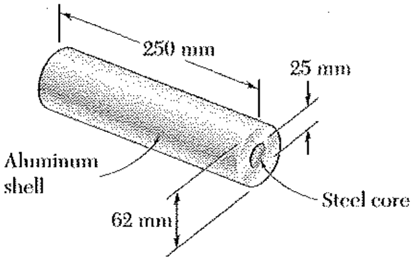
Total force. $P = 2P_a + P_b = 3.5 P_a \qquad P_a = \frac{2}{7} P$

$$\delta = -\frac{P_a L}{E_a A} = -\frac{2}{7}\frac{PL}{E_a A} = -\frac{2}{7}\frac{(45 \times 10^3)(250 \times 10^{-3})}{(70 \times 10^9)(150 \times 10^{-6})} = -306 \times 10^{-6} \text{ m}$$

$\delta = -0.306 \text{ mm}$ ◄

Problem 2.35

2.35 Compressive centric forces of 160 kN are applied at both ends of the assembly shown by means of rigid plates. Knowing that $E_s = 200$ GPa and $E_a = 70$ GPa, determine (*a*) the normal stresses in the steel core and the aluminum shell, (*b*) the deformation of the assembly.



Let P_a = portion of axial force carried by shell.

P_s = portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s}{L}\delta$$

Total force $\quad P = P_a + P_s = (E_a A_a + E_s A_s)\frac{\delta}{L}$

$$\frac{\delta}{L} = \varepsilon = \frac{P}{E_a A_a + E_s A_s}$$

Data: $P = 160$ kN

$$A_a = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(0.062^2 - 0.025^2) = 0.002528 \text{ m}$$

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.025)^2 = 0.000491 \text{ m}^2$$

$$\varepsilon = \frac{-160000}{(70\times10^9)(0.002528) + (200\times10^9)(0.000491)} = -581.5\times10^{-6}$$

(a) $\sigma_s = E_s\varepsilon = (200\times10^9)(-581.5\times10^{-6}) = -116.3 \text{ MPa}$

$\sigma_a = E_a\varepsilon = (70\times10^9)(-581.5\times10^{-6}) = -40.7 \text{ MPa}$.

(b) $\delta = L\varepsilon = (0.25)(-581.5\times10^{-6}) = -145\times10^{-6} \text{ m} = -0.145 \text{ mm}$.

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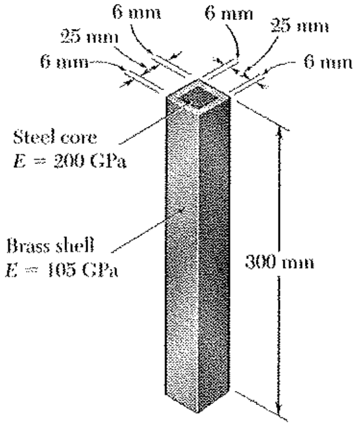
Problem 2.36

2.36 The length of the assembly decreases by 0.15 mm when an axial force is applied by means of rigid end plates. Determine (*a*) the magnitude of the applied force, (*b*) the corresponding stress in the steel core.



Let P_b = portion of axial force carried by brass shell.

P_s = portion of axial force carried by steel core.

$$\delta = \frac{P_b L}{A_b E_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_s L}{A_s E_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_b + P_s = (E_b A_b + E_s A_s)\frac{\delta}{L}$$

$$A_s = (0.025)^2 = 625 \times 10^{-6}\ m^2.$$

$$A_b = (0.037)^2 - (0.025)^2 = 744 \times 10^{-6}\ m^2.$$

(a) $P = [(105\times10^9)(744\times10^{-6}) + (200\times10^9)(625\times10^{-6})]\frac{0.00015}{0.3} = 101.6\ kN$

(b) $\sigma_s = E_s \varepsilon = E_s \frac{\delta}{L} = (200\times10^9)\frac{0.00015}{0.3} = 100\ MPa.$

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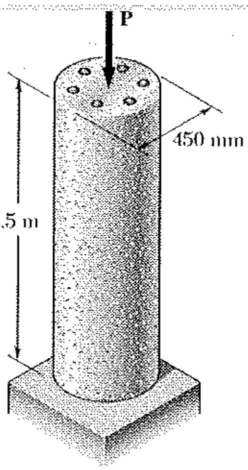
Problem 2.37

2.37 The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200$ GPa and $E_c = 25$ GPa, determine the normal stresses in the steel and in the concrete when a 1550 kN axial centric force **P** is applied to the post.



Let P_c = portion of axial force carried by concrete.

P_s = portion carried by the six steel rods.

$$\delta = \frac{P_c L}{E_c A_c} \qquad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{P}{E_c A_c + E_s A_s}$$

$$A_s = 6\,\frac{\pi}{4} d_s^2 = \frac{6\pi}{4}(28)^2 = 3.6945 \times 10^3\ \text{mm}^2 = 3.6945 \times 10^{-3}\ \text{m}^2$$

$$A_c = \frac{\pi}{4} d_c^2 - A_s = \frac{\pi}{4}(450)^2 - 3.6945 \times 10^3 = 155.349 \times 10^3\ \text{mm}^2$$

$$= 153.349 \times 10^{-3}\ \text{m}^2$$

$$L = 1.5\ \text{m}$$

$$\varepsilon = \frac{1550 \times 10^3}{(25\times10^9)(155.349\times10^{-3}) + (200\times10^9)(3.6945\times10^{-3})} = 335.31 \times 10^{-6}$$

$$\sigma_s = E_s \varepsilon = (200\times10^9)(335.31\times10^{-6}) = 67.1 \times 10^6\ \text{Pa} = 67.1\ \text{MPa}$$ ◄

$$\sigma_c = E_c \varepsilon = (25\times10^9)(335.31\times10^{-6}) = 8.38 \times 10^6\ \text{Pa} = 8.38\ \text{MPa}$$ ◄

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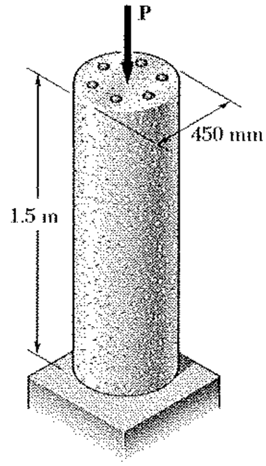
Problem 2.38

2.38 For the post of Prob. 2.37, determine the maximum centric force which can be applied if the allowable normal stress is 160 MPa in the steel and 18 MPa in the concrete.



2.37 The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200$ GPa and $E_c = 25$ GPa, determine the normal stresses in the steel and in the concrete when a 1550 kN axial centric force **P** is applied to the post.

Determine allowable strain in each material.

Steel: $\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{160\times10^6}{200\times10^9} = 800\times10^{-6}$

Concrete: $\varepsilon_c = \frac{\sigma_c}{E_c} = \frac{18\times10^6}{25\times10^9} = 720\times10^{-6}$

Smaller value governs $\varepsilon = \frac{\delta}{L} = 720\times10^{-6}$

Let P_c = portion of load carried by concrete.

P_s = portion carried by six steel rods.

$$\delta = \frac{P_c L}{E_c A_c}, \quad P_c = E_c A_c \frac{\delta}{L} = E_c A_c \varepsilon$$

$$\delta = \frac{P_s L}{E_s A_s}, \quad P_s = E_s A_s \frac{\delta}{L} = E_s A_s \varepsilon$$

$$P = P_c + P_s = (E_c A_c + E_s A_s)\varepsilon$$

$$A_s = 6\frac{\pi}{4}d_s^2 = \frac{6\pi}{4}(28)^2 = 3.6945\times10^3 \text{ mm}^2 = 3.6945\times10^{-3} \text{ m}^2$$

$$A_c = \frac{\pi}{4}d_c^2 - A_s = \frac{\pi}{4}(450)^2 - 3.6945\times10^3 = 155.349\times10^3 \text{ mm}^2$$

$$= 155.349\times10^{-3} \text{ m}^2$$

$$P = [(25\times10^9)(155.349\times10^{-3}) + (200\times10^9)(3.6945\times10^{-3})](720\times10^{-6})$$

$$= 3.33\times10^6 \text{ N} \qquad 3330 \text{ kN} \blacktriangleleft$$

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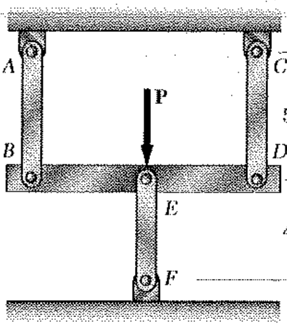
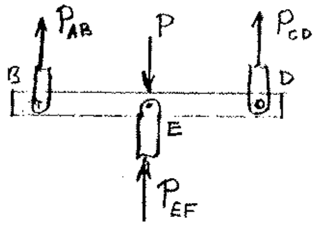
Problem 2.39



2.39 Three steel rods ($E = 200$ GPa) support a 36-kN load **P**. Each of the rods AB and CD has a 200-mm^2 cross-sectional area and rod EF has a 625-mm^2 cross-sectional area. Neglecting the deformation of rod BED, determine (*a*) the change in length of rod EF, (*b*) the stress in each rod.

Use member BED as a free body.



By symmetry, or by $\Sigma M_E = 0$

$$P_{CD} = P_{AB}$$

$\Sigma F_y = 0$:

$$P_{AB} + P_{CD} + P_{EF} - P = 0$$

$$P = 2P_{AB} + P_{EF}$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{EA_{AB}}, \quad \delta_{CD} = \frac{P_{CD}L_{CD}}{EA_{CD}}, \quad \delta_{EF} = \frac{P_{EF}L_{EF}}{EA_{EF}}$$

Since $L_{AB} = L_{CD}$ and $A_{AB} = A_{CD}$, $\delta_{AB} = \delta_{CD}$

Since points A, C, and E are fixed, $\delta_B = \delta_{AB}$, $\delta_D = \delta_{CD}$, $\delta_E = \delta_{EF}$

Since member BED is rigid, $\delta_E = \delta_B = \delta_C$

$$\frac{P_{AB}L_{AB}}{EA_{AB}} = \frac{P_{EF}L_{EF}}{EA_{EF}} \quad \therefore \quad P_{AB} = \frac{A_{AB}}{A_{EF}} \cdot \frac{L_{EF}}{L_{AB}} P_{EF} = \frac{200}{625} \cdot \frac{400}{500} P_{EF}$$

$$= 0.256\ P_{EF}$$

$$P = 2P_{AB} + P_{EF} = (2)(0.256)P_{EF} + P_{EF} = 1.512\ P_{EF}$$

$$P_{EF} = \frac{P}{1.512} = \frac{36 \times 10^3}{1.512} = 23.810 \times 10^3 \text{ N}$$

$$P_{AB} = P_{CD} = (0.256)(23.810 \times 10^3) = 6.095 \times 10^3 \text{ N}$$

(a) $$\delta = \delta_{EF} = \frac{(23.810 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(625 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$$

$$= 0.0762 \text{ mm} ◀$$

or

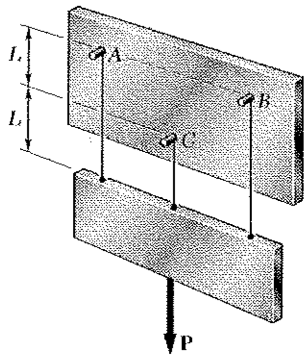
$$\delta = \delta_{AB} = \frac{(6.095 \times 10^3)(500 \times 10^{-3})}{(200 \times 10^9)(200 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$$

(b) $$\sigma_{AB} = \sigma_{CD} = \frac{P_{AB}}{A_{AB}} = \frac{6.095 \times 10^3}{200 \times 10^{-6}} = 30.5 \times 10^6 \text{ Pa} = 30.5 \text{ MPa} ◀$$

$$\sigma_{EF} = -\frac{P_{EF}}{A_{EF}} = -\frac{23.810 \times 10^3}{625 \times 10^{-6}} = -38.1 \times 10^6 \text{ Pa} = 38.1 \text{ MPa} ◀$$

Problem 2.40

2.40 Three wires are used to suspend the plate shown. Aluminum wires are used at A and B with a diameter of 3 mm and a steel wire is used at C with a diameter of 2 mm. Knowing that the allowable stress for aluminum (E = 70 GPa) is 98 MPa and that the allowable stress for steel (E = 200 GPa) is 126 MPa, determine the maximum load **P** that can be applied.



By symmetry $P_A = P_B$, and $\delta_A = \delta_B$

Also, $\delta_C = \delta_A = \delta_B = \delta$

Strain in each wire.

$$\varepsilon_A = \varepsilon_B = \frac{\delta}{2L}, \qquad \varepsilon_C = \frac{\delta}{L} = 2\,\varepsilon_A$$

Determine allowable strain.

Wires A & B: $\varepsilon_A = \dfrac{\sigma_A}{E_A} = \dfrac{98}{70\times10^3} = 1.4\times10^{-3}$

$\varepsilon_c = 2\,\varepsilon_A = 4.8\times10^{-3}$

Wire C: $\varepsilon_C = \dfrac{\sigma_c}{E_c} = \dfrac{126}{200\times10^3} = 0.63\times10^{-3}$

$\varepsilon_A = \varepsilon_B = \tfrac{1}{2}\varepsilon_C = 0.315\times10^{-3}$

Allowable strain for wire C governs ∴ $\sigma_c = 126\text{ MPa}$

$\sigma_A = E_A\varepsilon_A \qquad P_A = A_A E_A \varepsilon_A = \frac{\pi}{4}(0.003)^2(70\times10^9)(0.315\times10^{-3})$

$= 155.87\text{ N}$

$P_B = 155.87\text{N}$

$\sigma_c = E_c\varepsilon_c \qquad P_c = A_c.\sigma_c = \frac{\pi}{4}(0.002)^2(126\times10^6) = 395.84\text{ N}$

For equilibrium of the plate,

$P = P_A + P_B + P_c = 707.6\text{ N}$ ◄

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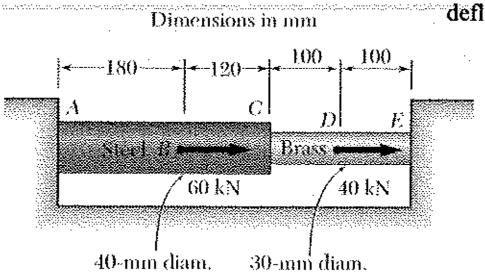
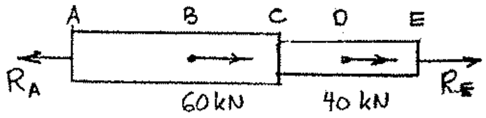
Problem 2.41

2.41 Two cylindrical rods, one of steel and the other of brass, are joined at *C* and restrained by rigid supports at *A* and *E*. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at *A* and *E*, (*b*) the deflection of point *C*.





A to C: $E = 200 \times 10^9$ Pa

$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$

$EA = 251.327 \times 10^6$ N

C to E: $E = 105 \times 10^9$ Pa

$A = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$

$EA = 74.220 \times 10^6$ N

A to B: $P = R_A$ $\qquad L = 180 \text{ mm} = 0.180 \text{ m}$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327 \times 10^6} = 716.20 \times 10^{-12} R_A$$

B to C: $P = R_A - 60 \times 10^3$ $\qquad L = 120 \text{ mm} = 0.120 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{251.327 \times 10^6} = 447.47 \times 10^{-12} R_A - 26.848 \times 10^{-6}$$

C to D: $P = R_A - 60 \times 10^3$ $\qquad L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 80.841 \times 10^{-6}$$

D to E: $P = R_A - 100 \times 10^3$ $\qquad L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 134.735 \times 10^{-6}$$

A to E: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6}$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(a) $3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6} = 0$ $\qquad R_A \doteq 62.831 \times 10^3$ N $\qquad$ 62.8 kN ← ◄

$R_E = R_A - 100 \times 10^3 = 62.8 \times 10^3 - 100 \times 10^3 = -37.2 \times 10^3$ N $\qquad$ 37.2 kN ← ◄

(b) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367 \times 10^{-9} R_A - 26.848 \times 10^{-6}$

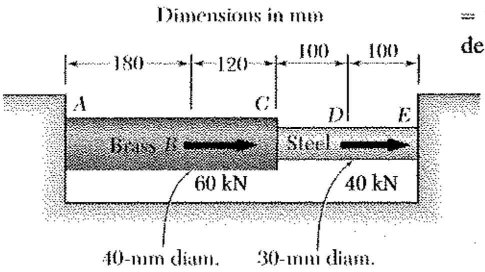
$= (1.16369 \times 10^{-9})(62.831 \times 10^3) - 26.848 \times 10^{-6}$

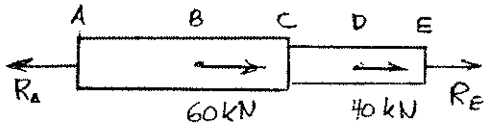
$= 46.3 \times 10^{-6}$ m $\qquad \delta_c = 46.3\ \mu\text{m}$ → ◄

Problem 2.42

2.42 Solve Prob. 2.41, assuming that rod AC is made of brass and rod CE is made of steel.

2.41 Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at A and E, (*b*) the deflection of point C.





A to C: $E = 105 \times 10^9$ Pa

$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$

$EA = 131.947 \times 10^6$ N

C to E: $E = 200 \times 10^9$ Pa

$A = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$

$EA = 141.372 \times 10^6$ N

A to B: $P = R_A$ $\quad L = 180 \text{ mm} = 0.180 \text{ m}$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{131.947 \times 10^6} = 1.36418 \times 10^{-9} R_A$$

B to C: $P = R_A - 60 \times 10^3$ $\quad L = 120 \text{ mm} = 0.120 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{131.947 \times 10^6} = 909.456 \times 10^{-12} R_A - 54.567 \times 10^{-6}$$

C to D: $P = R_A - 60 \times 10^3$ $\quad L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{141.372 \times 10^6} = 707.354 \times 10^{-12} R_A - 42.441 \times 10^{-6}$$

D to E: $P = R_A - 100 \times 10^3$ $\quad L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{141.372 \times 10^6} = 707.354 \times 10^{-12} R_A - 70.735 \times 10^{-6}$$

A to E: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 3.68834 \times 10^{-9} R_A - 167.743 \times 10^{-6}$

Since point E cannot move relative to A, $\quad \delta_{AE} = 0$

(a) $3.68834 \times 10^{-9} R_A - 167.743 \times 10^{-6} = 0 \quad R_A = 45.479 \times 10^3$ N $\quad$ 45.5 kN ← ◀

$R_E = R_A - 100 \times 10^3 = 45.479 \times 10^3 - 100 \times 10^3 = -54.521 \times 10^3$ $\quad$ 54.5 kN ← ◀

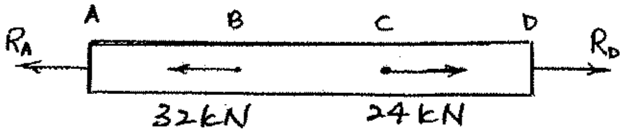
(b) $\delta_C = \delta_{AB} + \delta_{BC} = 2.27364 \times 10^{-9} R_A - 54.567 \times 10^{-6}$

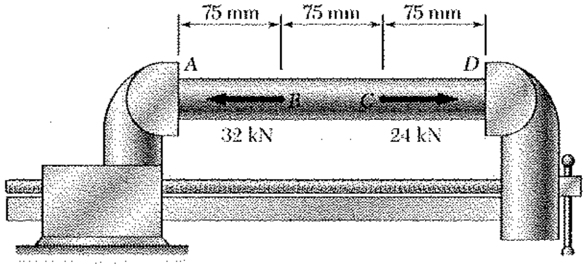
$= (2.27364 \times 10^{-9})(45.479 \times 10^3) - 54.567 \times 10^{-6}$

$= 48.8 \times 10^{-6}$ m $\quad \delta_C = 48.8\ \mu\text{m}$ → ◀

Problem 2.43

2.43 A steel tube (E_s = 200 GPa) with a 30-mm outer diameter and a 3-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (*a*) the forces exerted by the vise on the tube at *A* and *D*, (*b*) the change in length of the portion *BC* of the tube.





For the tube $d_o = 30$ mm

$$d_i = d_o - 2t = 0.03 - 2(0.003) = 0.024 \text{ m}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(0.03^2 - 0.024^2) = 254.5\times10^{-6}\text{ m}^2$$

A to B: $P = R_A$ kN $\quad L = 0.075$ m

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.075)}{(200\times10^9)(254.5\times10^{-6})} = 1.473\times10^{-9} R_A \text{ m}$$

B to C: $P = R_A + 32$ kN, $L = 0.075$ m.

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A+32000)(0.075)}{(200\times10^9)(254.5\times10^{-6})} = 1.473\times10^{-9}R_A + 47.15\times10^{-6}\text{ m}$$

C to D: $P = R_A + 8$ kN, $L = 0.075$ m

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A+8000)(0.075)}{(200\times10^9)(254.5\times10^{-6})} = 1.473\times10^{-9}R_A + 11.79\times10^{-6}$$

A to D: $\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 4.419\times10^{-9}R_A + 58.94\times10^{-6}$ m

Given jaw movement $\quad \delta_{AD} = -0.0002$ m

(a) $4.419\times10^{-9}R_A + 58.94\times10^{-6} = -0.0002$

$R_A = -58597$ N $\qquad R_A = 58.6$ kN →

$R_D = R_A + 8000 = -50597$ N $\qquad R_D = 50.6$ kN ←

(b) $\delta_{BC} = (1.473\times10^{-9})(-58597) + 47.15\times10^{-6} = 39.2\times10^{-6}$ m ◄

$= 39.2\times10^{-3}$ mm

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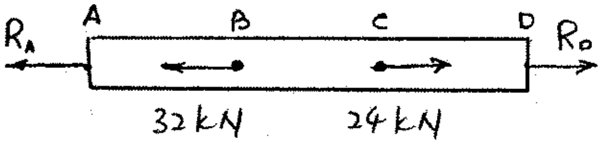
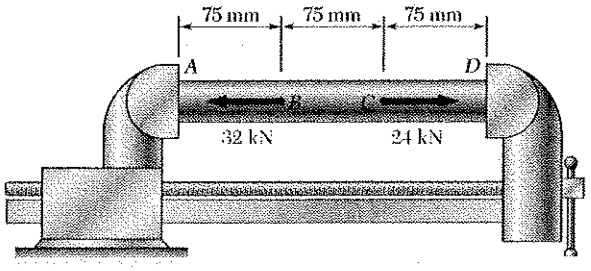
Problem 2.44

2.44 Solve Prob. 2.43, assuming that after the forces have been applied, the vise is adjusted to increase the distance between its jaws by 0.1 mm.

2.43 A steel tube (E_s = 200 GPa) with a 30-mm outer diameter and a 3-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (*a*) the forces exerted by the vise on the tube at *A* and *D*, (*b*) the change in length of the portion *BC* of the tube.





For the tube $d_o = 30$ mm

$d_i = d_o - 2t = 0.03 - 2(0.003) = 0.024$ m

$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(0.03^2 - 0.024^2) = 254.5 \times 10^{-6}$ m^2

A to B: $P = R_A$ $\qquad L = 0.075$ m

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.075)}{(200\times10^9)(254.5\times10^{-6})} = 1.473\times10^{-9} R_A \text{ m.}$$

B to C: $P = R_A + 32$ kN, $\qquad L = 0.075$ m

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A+32000)(0.075)}{(200\times10^9)(254.5\times10^{-6})} = 1.473\times10^{-9} R_A + 47.15\times10^{-6} \text{ m}$$

C to D: $P = R_A + 8000$ N $\qquad L = 0.075$ m.

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A+8000)(0.075)}{(200\times10^9)(254.5\times10^{-6})} = 1.473\times10^{-9} R_A + 11.79\times10^{-6} \text{ m}$$

A to D: $\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 4.419\times10^{-9} R_A + 58.94\times10^{-6}$

Given jaw movement $\delta_{AD} = -0.0001$ m

(a) $4.419\times10^{-9} R_A + 58.94\times10^{-6} = -0.0001$

$R_A = -35967$ N $\qquad R_A = 35.97$ kN → ◀

$R_D = R_A + 8000 = -27967$ $\qquad R_D = 27.97$ kN ← ◀

(b) $\delta_{BC} = (1.473\times10^{-9})(-35967) + 47.15\times10^{-6} = -5.83\times10^{-6}$ m ◀

$= -5.83\times10^{-3}$ mm

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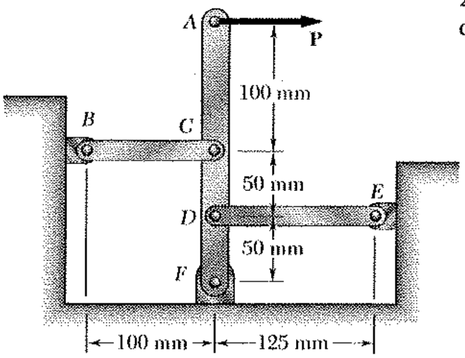
Problem 2.45

2.45 Links *BC* and *DE* are both made of steel (E = 200 GPa) and are 12 mm wide and 6 mm thick. Determine (*a*) the force in each link when a 2.4-kN force **P** is applied to the rigid member *AF* shown, (*b*) the corresponding deflection of point *A*.



Let the rigid member ACDF rotate through small angle θ clockwise about point F.

Then $\delta_C = \delta_{BC} = 0.1\,\theta\ m \rightarrow$

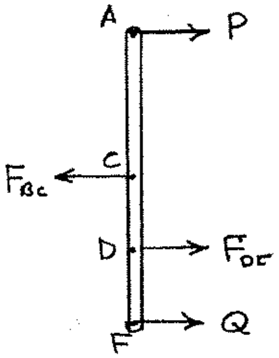
$\delta_D = -\delta_{DE} = 0.05\,\theta\ m \rightarrow$

$$\delta = \frac{FL}{EA} \quad \text{or} \quad F = \frac{EA\delta}{L}$$

For links: $A = (0.012)(0.006) = 72\times10^{-6}\ m^2$, $L_{BC} = 0.1\ m$, $L_{DE} = 0.125\ m$

$$F_{BC} = \frac{EA\,\delta_{BC}}{L_{BC}} = \frac{(200\times10^9)(72\times10^{-6})(0.1\theta)}{0.1} = 14.4\times10^6\ \theta$$

$$F_{DE} = \frac{EA\,\delta_{DE}}{L_{DE}} = \frac{(200\times10^9)(72\times10^{-6})(-0.05\theta)}{0.125} = -5.76\times10^6\ \theta$$



Use member ACDF as a free body.

$$\circlearrowright\!+\ \Sigma M_F = 0:\ 0.2P - 0.1F_{BC} + 0.05F_{DE} = 0$$

$$P = \tfrac{1}{2}F_{BC} - \tfrac{1}{4}F_{DE}$$

$$2400 = \tfrac{1}{2}(14.4\times10^6)\theta - \tfrac{1}{4}(-5.76\times10^6)\theta = 8.64\times10^6\ \theta$$

$$\theta = 0.27778\times10^{-3}\ \text{rad}\ \circlearrowright$$

(a) $F_{BC} = (14.4\times10^6)(0.27778\times10^{-3})$ $\qquad F_{BC} = 4\ kN$ ◄

$F_{DE} = -(5.76\times10^6)(0.27778\times10^{-3})$ $\qquad F_{DE} = -1.6\ kN$ ◄

(b) Deflection at point A.

$\delta_A = 0.2\theta = (0.2)(0.27778\times10^{-3})$ $\qquad \delta_A = 0.056\ mm \rightarrow$ ◄

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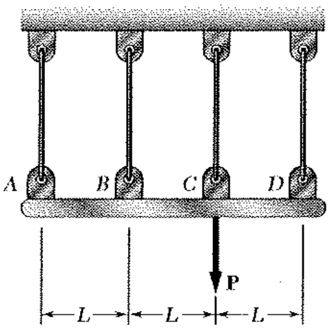
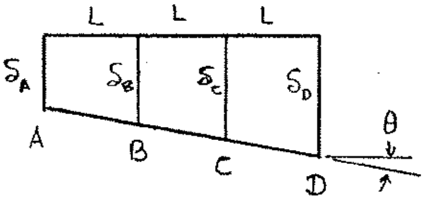
Problem 2.46

2.46 The rigid bar *ABCD* is suspended from four identical wires. Determine the tension in each wire caused by the load **P** shown.



Deformations Let θ be the rotation of bar ABCD and δ_A, δ_B, δ_C and δ_D be the deformations of wires A, B, C, and D.



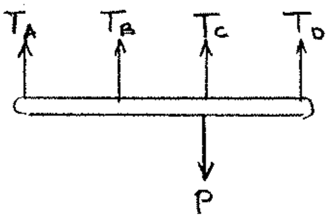
From geometry, $\theta = \dfrac{\delta_B - \delta_A}{L}$

$$\delta_B = \delta_A + L\theta$$

$$\delta_C = \delta_A + 2L\theta = 2\delta_B - \delta_A \qquad (1)$$

$$\delta_D = \delta_A + 3L\theta = 3\delta_B - 2\delta_A \qquad (2)$$

Since all wires are identical, the forces in the wires are proportional to the deformations.



$$T_C = 2T_B - T_A \qquad (1')$$

$$T_D = 3T_B - 2T_A \qquad (2')$$

Use bar ABCD as a free body.

$$+\circlearrowleft \Sigma M_C = 0: \quad -2LT_A - LT_B + LT_D = 0 \qquad (3)$$

$$+\uparrow \Sigma F_y = 0: \quad T_A + T_B + T_C + T_D - P = 0 \qquad (4)$$

Substituting (2′) into (3) and dividing by L,

$$-4T_A + 2T_B = 0 \qquad T_B = 2T_A \qquad (3')$$

Substituting (1′), (2′), and (3′) into (4),

$$T_A + 2T_A + 3T_A + 4T_A - P = 0 \qquad 10T_A = P$$

$$T_A = \tfrac{1}{10}P \quad ◄$$

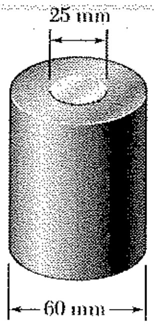
$$T_B = 2T_A = (2)(\tfrac{1}{10})P \qquad T_B = \tfrac{1}{5}P \quad ◄$$

$$T_C = (2)(\tfrac{1}{5}P) - (\tfrac{1}{10}P) \qquad T_C = \tfrac{3}{10}P \quad ◄$$

$$T_D = (3)(\tfrac{1}{5}P) - (2)(\tfrac{1}{10}P) \qquad T_D = \tfrac{2}{5}P \quad ◄$$

Problem 2.47

2.47 The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15 °C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195 °C.



Brass core
$E = 105$ GPa
$\alpha = 20.9\times10^{-6}/°C$

Aluminum shell
$E = 70$ GPa
$\alpha = 23.6\times10^{-6}/°C$

Let L be the length of the assembly.

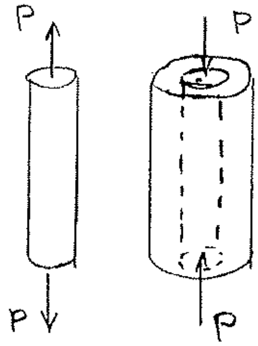
Free thermal expansion.

$\Delta T = 195 - 15 = 180°C$

Brass core: $(\delta_T)_b = L\alpha_b(\Delta T)$

Aluminum shell: $(\delta_T) = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core. $\delta = L(\alpha_a - \alpha_b)(\Delta T)$



Let P be the tensile force in the core and the compressive force in the shell.

Brass core: $E_b = 105\times10^9$ Pa

$A_b = \frac{\pi}{4}(25)^2 = 490.87\ \text{mm}^2 = 490.87\times10^{-6}\ \text{m}^2$

$(\delta_P)_b = \dfrac{PL}{E_bA_b}$

Aluminum shell: $E_a = 70\times10^9$ Pa

$A_a = \frac{\pi}{4}(60^2 - 25^2) = 2.3366\times10^3\ \text{mm}^2 = 2.3366\times10^{-3}\ \text{m}^2$

$\delta = (\delta_P)_b + (\delta_P)_a$

$L(\alpha_b - \alpha_a)(\Delta T) = \dfrac{PL}{E_bA_b} + \dfrac{PL}{E_aA_a} = KPL$

where $K = \dfrac{1}{E_bA_b} + \dfrac{1}{E_aA_a} = \dfrac{1}{(105\times10^9)(490.87\times10^{-6})} + \dfrac{1}{(70\times10^9)(2.3366\times10^{-3})}$

$= 25.516\times10^{-9}\ N^{-1}$

Then $P = \dfrac{(\alpha_b - \alpha_a)(\Delta T)}{K} = \dfrac{(23.6\times10^{-6} - 20.9\times10^{-6})(180)}{25.516\times10^{-9}} = 19.047\times10^3\ N$

Stress in aluminum. $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{19.047\times10^3}{2.3366\times10^{-3}} = -8.15\times10^6$ Pa

$\sigma_a = -8.15$ MPa ◄

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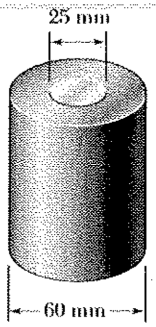
Problem 2.48

2.48 Solve Prob. 2.47, assuming that the core is made of steel ($E = 200$ GPa, $\alpha = 11.7 \times 10^{-6}/°C$) instead of brass.



2.47 The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15 °C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195 °C.

Aluminum shell
$E = 70$ GPa
$\alpha = 23.6 \times 10^{-6}/°C$

Let L be the length of the assembly.

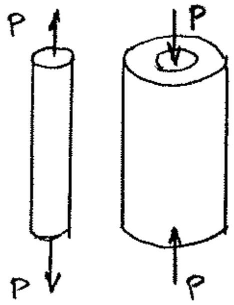
Free thermal expansion.

$$\Delta T = 195 - 15 = 180°C$$

Steel core: $(\delta_T)_s = L\alpha_s(\Delta T)$

Aluminum shell: $(\delta_T)_a = L\alpha_a(\Delta T)$

Net expension of shell with respect to the core. $\delta = L(\alpha_a - \alpha_s)(\Delta T)$



Let P be the tensile force in the core and the compressive force in the shell.

Steel core: $E_s = 200 \times 10^9$ Pa

$$A_s = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$(\delta_P)_s = \frac{PL}{E_s A_s}$$

Aluminum shell: $E_a = 70 \times 10^9$ Pa $\qquad (\delta_P)_a = \frac{PL}{E_a A_a}$

$$A_a = \frac{\pi}{4}(60^2 - 25)^2 = 2.3366 \times 10^3 \text{ mm}^2 = 2.3366 \times 10^{-3} \text{ m}^2$$

$$\delta = (\delta_P)_s + (\delta_P)_a$$

$$L(\alpha_a - \alpha_s)(\Delta T) = \frac{PL}{E_s A_s} + \frac{PL}{E_a A_a} = KPL$$

where $K = \frac{1}{E_s A_s} + \frac{1}{E_a A_a} = \frac{1}{(200 \times 10^9)(490.87 \times 10^{-6})} + \frac{1}{(70 \times 10^9)(2.3366 \times 10^{-3})}$

$$= 16.2999 \times 10^{-9} \text{ N}^{-1}$$

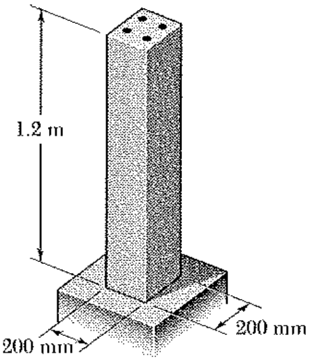
Then $P = \frac{(\alpha_a - \alpha_s)(\Delta T)}{K} = \frac{(23.6 \times 10^{-6} - 11.7 \times 10^{-6})(180)}{16.2999 \times 10^{-9}} = 131.41 \times 10^3$ N

Stress in aluminum. $\sigma_a = -\frac{P}{A_a} = -\frac{131.19 \times 10^3}{2.3366 \times 10^{-3}} = -56.2 \times 10^6$ Pa

$\sigma_a = -56.2$ MPa ◄

Problem 2.49

2.49 A 1.2-m concrete post is reinforced by four steel bars, each of 18 mm diameter. Knowing that $E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}$/°C and $E_c = 25$ GPa and $\alpha_c = 9.9 \times 10^{-6}$/°C , determine the normal stresses induced in the steel and in the concrete by a temperature rise of 27°C.



$$A_s = 4\,\frac{\pi}{4}d^2 = (4)(\tfrac{\pi}{4})(0.018)^2 = 0.0010179 \text{ m}^2.$$

$$A_c = A - A_s = 0.2^2 - 0.0010179 = 0.039 \text{ m}^2.$$

Let P_c be the tensile in the concrete. For equilibrium with zero total force, the compressive force in the four steel rods is $-P_c$.

Strains: $$\varepsilon_s = \frac{P_s}{E_sA_s} + \alpha_s(\Delta T) = -\frac{P_c}{E_sA_s} + \alpha_s(\Delta T)$$

$$\varepsilon_c = \frac{P_c}{E_cA_c} + \alpha_c(\Delta T)$$

Matching: $$\varepsilon_c = \varepsilon_s \qquad \frac{P_c}{E_cA_c} + \alpha_c(\Delta T) = -\frac{P_c}{E_sA_s} + \alpha_s(\Delta T)$$

$$\left(\frac{1}{E_cA_c} + \frac{1}{E_sA_s}\right)P_c = (\alpha_s - \alpha_c)(\Delta T)$$

$$\left(\frac{1}{(25\times10^9)(0.039)} + \frac{1}{(200\times10^9)(0.0010179)}\right)P_c$$

$$= (11.7 - 9.9)(10^{-6})(27)$$

$P_c = 8185$ N. $\qquad P_s = -8185$ N.

Stress in steel $\quad \sigma_s = \dfrac{P_s}{A_s} = \dfrac{-8.185\times10^3}{0.0010179} = -8.04$ MPa

Stress in concrete $\quad \sigma_c = \dfrac{P_c}{A_c} = \dfrac{8.185\times10^3}{0.039} = 0.21$ MPa.

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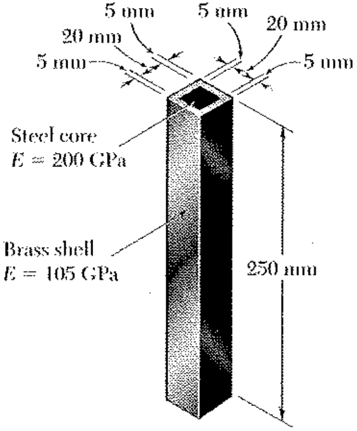
Problem 2.50

2.50 The brass shell ($\alpha_b = 20.9 \times 10^{-6}/°\text{C}$) is fully bonded to the steel core ($\alpha_s = 11.7 \times 10^{-6}/°\text{F}$). Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 55 MPa.



Let P_s = axial force developed in the steel core

For equilibrium with zero total force the compressive force in the brass shell is P_s.

Strains $\quad \varepsilon_s = \dfrac{P_s}{E_s A_s} + \alpha_s(\Delta T)$

$$\varepsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

Matching, $\varepsilon_s = \varepsilon_b$

$$\frac{P_s}{E_s A_s} + \alpha_s(\Delta T) = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

$$\left(\frac{1}{E_s A_s} + \frac{1}{E_b A_b}\right) P_s = (\alpha_b - \alpha_s)(\Delta T)$$

$$A_s = (0.020)(0.020) = 400 \times 10^{-6} \text{ m}^2$$

$$A_b = (0.030)(0.030) - (0.020)(0.020) = 500 \times 10^{-6} \text{ m}^2$$

$$\alpha_b - \alpha_s = 9.2 \times 10^{-6} /°\text{C}$$

$$P_s = \sigma_s A_s = (55 \times 10^6)(400 \times 10^{-6}) = 22 \times 10^3 \text{ N}$$

$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(200 \times 10^9)(400 \times 10^{-6})} + \frac{1}{(105 \times 10^9)(500 \times 10^{-6})} = 31.55 \times 10^{-9} \text{ N}^{-1}$$

$$(31.55 \times 10^{-9})(22 \times 10^3) = (9.2 \times 10^{-6})(\Delta T)$$

$$\Delta T = 75.4\ °\text{C} \blacktriangleleft$$

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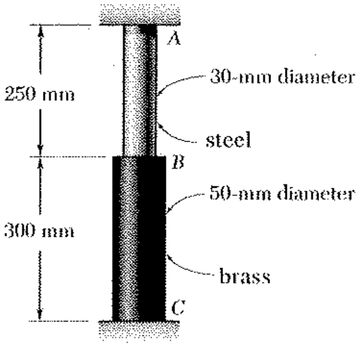
Problem 2.51

2.51 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$) and portion BC is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°\text{C}$). Knowing that the rod is initially unstressed, determine the compressive force induced in ABC when there is a temperature rise of 50°C.

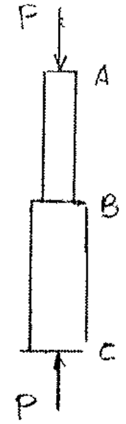


$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(50)^2 = 1.9635 \times 10^3 \text{ mm}^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

Free thermal expansion.

$$\delta_T = L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)$$
$$= (0.250)(11.7\times10^{-6})(50) + (0.300)(20.9\times10^{-6})(50)$$
$$= 459.75 \times 10^{-6} \text{ m}$$



Shortening due to induced compressive force P.

$$\delta_P = \frac{PL}{E_s A_{AB}} + \frac{PL}{E_b A_{BC}}$$
$$= \frac{0.250\,P}{(200\times10^9)(706.86\times10^{-6})} + \frac{0.300\,P}{(105\times10^9)(1.9635\times10^{-3})}$$
$$= 3.2235 \times 10^{-9}\, P$$

For zero net deflection, $\delta_P = \delta_T$

$$3.2235\times10^{-9}\, P = 459.75\times10^{-6}$$

$$P = 142.62\times10^3 \text{ N} \qquad P = 142.6 \text{ kN} \blacktriangleleft$$

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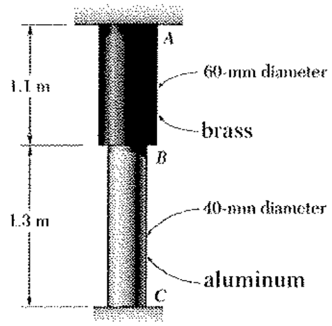
Problem 2.52

2.52 A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°\text{C}$) and portion *BC* is made of aluminum ($E_s = 72$ GPa, $\alpha_s = 23.9 \times 10^{-6}/°\text{C}$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 42°C, (*b*) the corresponding deflection of point *B*.



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(60)^2 = 2.8274 \times 10^3 \text{ mm}^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

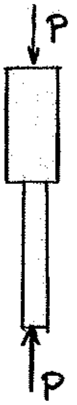
$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(40)^2 = 1.2566 \times 10^3 \text{ mm}^2 = 1.2566 \times 10^{-3} \text{ m}^2$$

Free thermal expansion

$$\delta_T = L_{AB}\alpha_b(\Delta T) + L_{BC}\alpha_a(\Delta T)$$

$$= (1.1)(20.9 \times 10^{-6})(42) + (1.3)(23.9 \times 10^{-6})(42)$$

$$= 2.2705 \times 10^{-3} \text{ m}$$



Shortening due to induced compressive force

$$\delta_P = \frac{PL_{AB}}{E_b A_{AB}} + \frac{PL_{BC}}{E_a A_{BC}}$$

$$= \frac{1.1\ P}{(105 \times 10^9)(2.8274 \times 10^{-3})} + \frac{1.3\ P}{(72 \times 10^9)(1.2566 \times 10^{-3}}$$

$$= 18.074 \times 10^{-9}\ P$$

For zero net deflection $\delta_P = \delta_T$

$$18.074 \times 10^{-9}\ P = 2.2705 \times 10^{-3}$$

$$P = 125.62 \times 10^3 \text{ N}$$

(a) $\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{125.62 \times 10^3}{2.8274 \times 10^{-3}} = -44.4 \times 10^6 \text{ Pa} = -44.4 \text{ MPa}$ ◀

$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{125.62 \times 10^3}{1.2566 \times 10^{-3}} = -100.0 \times 10^6 \text{ Pa} = -100.0 \text{ MPa}$ ◀

(b) $\delta_B = +\frac{PL_{AB}}{E_b A_{AB}} - L_{AB}\alpha_b(\Delta T)$

$$= \frac{(125.62 \times 10^3)(1.1)}{(105 \times 10^9)(2.8274 \times 10^{-3})} - (1.1)(20.9 \times 10^{-6})(42)$$

$$= -500 \times 10^{-6} \text{ m} = -0.500 \text{ mm}$$

i.e. 0.500 mm ↓ ◀

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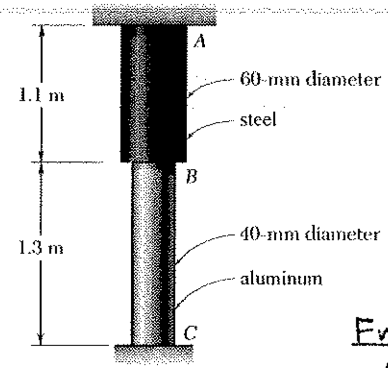
Problem 2.53

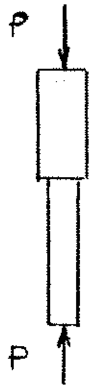
2.53 Solve Prob. 2.52, assuming that portion AB of the composite rod is made of steel and portion BC is made of brass.



2.52 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of brass (E_b = 105 GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) and portion BC is made of aluminum (E_a = 72 GPa, $\alpha_a = 23.9 \times 10^{-6}/°C$). Knowing that the rod is initially unstressed, determine (a) the normal stresses induced in portions AB and BC by a temperature rise of 42°C, (b) the corresponding deflection of point B.

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.06)^2 = 2.8274 \times 10^{-6}\ m^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.04)^2 = 1.2566 \times 10^{-6}\ m^2$$



Free thermal expansion. $\Delta T = 42°$

$$(\delta_T)_{AB} = L_{AB}\,\alpha_s(\Delta T) = (1.1)(11.7 \times 10^{-6})(42) = 0.541 \times 10^{-3}\ m$$

$$(\delta_T)_{BC} = L_{BC}\,\alpha_b(\Delta T) = (1.3)(23.9 \times 10^{-6})(42) = 1.305 \times 10^{-3}\ m$$

Total $\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 1.846 \times 10^{-3}\ m$

Shortening due to induced compressive force P.

$$(\delta_P)_{AB} = \frac{P L_{AB}}{E_s A_{AB}} = \frac{1.1\,P}{(200 \times 10^9)(2.8274 \times 10^{-6})} = 1945.25 \times 10^{-9}\,P$$

$$(\delta_P)_{BC} = \frac{P L_{BC}}{E_b A_{BC}} = \frac{1.3\,P}{(72 \times 10^9)(1.2566 \times 10^{-6})} = 14368.58 \times 10^{-9}\,P$$

Total $\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 16313.8 \times 10^{-9}\,P$

For zero net deflection $\delta_P = \delta_T$

$16313.8 \times 10^{-9} P = 1.846 \times 10^{-3}$ $P = 113.16\ N$

(a) $\sigma_{AB} = -\dfrac{P}{A_{AB}} = -\dfrac{113.16}{2.8274 \times 10^{-6}} = -40.02\ MPa$ $\sigma_{AB} = -40\ MPa$ ◄

$\sigma_{BC} = -\dfrac{P}{A_{BC}} = -\dfrac{113.16}{1.2566 \times 10^{-6}} = -90.05\ MPa$ $\sigma_{BC} = -90.1\ MPa$ ◄

(b) $(\delta_P)_{AB} = (1945.25 \times 10^{-9})(113.16) = 0.22 \times 10^{-3} m$

$\delta_B = (\delta_T)_{AB}\downarrow + (\delta_P)_{AB}\uparrow = 0.541 \times 10^{-3}\downarrow + 0.22 \times 10^{-3}\uparrow$

$\delta_B = 0.32\ mm$ ↓ ◄

or $(\delta_P)_{BC} = (14368.58 \times 10^{-9})(113.16) = 1.626 \times 10^{-3}\ m$

$\delta_B = (\delta_T)_{BC}\uparrow + (\delta_P)_{BC}\downarrow = 1.305 \times 10^{-3}\uparrow + 1.626 \times 10^{-3}\downarrow$

$\delta_B = 0.32\ mm\downarrow$ (checks)

Problem 2.54

2.54 A steel railroad track (E_s = 200 GPa, $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$) was laid out at a temperature of –1.0°C. Determine the normal stress in the rail when the temperature reaches 52°C, assuming that the rails (*a*) are welded to form a continuous track, (*b*) are 12 m long with 6-mm gaps between them.

$L = 12\text{ m}$

$\delta_T = L\alpha(\Delta T) = (12)(11.7\times10^{-6})(52 - -1) = 0.00744\text{ m.}$

$\delta_P = \frac{PL}{EA} = \frac{L}{E}\sigma = \frac{12}{200\times10^{9}}\sigma = 60\times10^{-12}\sigma$

(a) $\delta_P + \delta_T = 0$ $\quad 60\times10^{-12}\sigma + 0.00744 = 0$ $\quad \sigma = -124\text{ MPa}$ ◄

(b) $\delta_P + \delta_T = 0.006$ $\quad 60\times10^{-12}\sigma + 0.00744 = 0.006$ $\quad \sigma = -24\text{ MPa.}$ ◄

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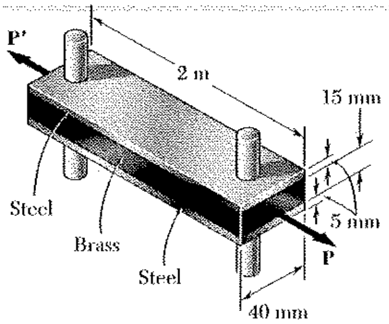
Problem 2.55



2.55 Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) which is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

(a) Required temperature change for fabrication.

$\delta_T = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$

Temperature change required to expand steel bar by this amount

$\delta_T = L\alpha_s \Delta T, \quad 0.5 \times 10^{-3} = (2.00)(11.7 \times 10^{-6})(\Delta T), \quad \Delta T =$

$0.5 \times 10^{-3} = (2)(11.7 \times 10^{-6})(\Delta T)$

$\Delta T = 21.368 \text{ °C} \qquad 21.4 \text{ °C} \blacktriangleleft$

(b) Once assembled, a tensile force P^* develops in the steel and a compressive force P^* develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$

$$(\delta_P)_s = \frac{F^* L}{A_s E_s} = \frac{P^*(2.00)}{(400 \times 10^{-6})(200 \times 10^9)} = 25 \times 10^{-9} P^*$$

Contraction of brass: $A_b = (40)(15) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$

$$(\delta_P)_b = \frac{P^* L}{A_b E_b} = \frac{P^*(2.00)}{(600 \times 10^{-6})(105 \times 10^9)} = 31.746 \times 10^{-9} P^*$$

But $(\delta_P)_s + (\delta_P)_b$ is equal to the initial amount of misfit

$(\delta_P)_s + (\delta_P)_b = 0.5 \times 10^{-3}, \quad 56.746 \times 10^{-9} P^* = 0.5 \times 10^{-3}$

$P^* = 8.811 \times 10^3 \text{ N}$

Stresses due to fabrication.

Steel: $\sigma_s^* = \dfrac{P^*}{A_s} = \dfrac{8.811 \times 10^3}{400 \times 10^{-6}} = 22.03 \times 10^6 \text{ Pa} = 22.03 \text{ MPa}$

Brass: $\sigma_b^* = -\dfrac{P^*}{A_b} = -\dfrac{8.811 \times 10^3}{600 \times 10^{-6}} = -14.68 \times 10^6 \text{ Pa} = -14.68 \text{ MPa}$

To these stresses must be added the stresses due to the 25 kN load.

continued

For the added load, the additional deformation is the same for both the steel and the brass. Let δ' be the additional displacement. Also, let P_s and P_b be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_s = \frac{A_s E_s}{L}\delta' = \frac{(400\times10^{-6})(200\times10^{9})}{2.00}\delta' = 40\times10^{6}\,\delta'$$

$$P_b = \frac{A_b E_b}{L}\delta' = \frac{(600\times10^{-6})(105\times10^{9})}{2.00}\delta' = 31.5\times10^{6}\,\delta'$$

Total $P = P_s + P_b = 25\times10^{3}\ \text{N}$

$$40\times10^{6}\,\delta' + 31.5\times10^{6}\,\delta' = 25\times10^{3} \qquad \delta' = 349.65\times10^{-6}\ \text{m}$$

$$P_s = (40\times10^{6})(349.65\times10^{-6}) = 13.986\times10^{3}\ \text{N}$$

$$P_b = (31.5\times10^{6})(349.65\times10^{-6}) = 11.140\times10^{3}\ \text{N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.986\times10^{3}}{400\times10^{-6}} = 34.97\times10^{6}\ \text{Pa}$$

$$\sigma_b = \frac{P_b}{A_b} = \frac{11.140\times10^{3}}{600\times10^{-6}} = 18.36\times10^{6}\ \text{Pa}$$

Add stress due to fabrication. Total stresses.

$$\sigma_s = 34.97\times10^{6} + 22.03\times10^{6} = 57.0\times10^{6}\ \text{Pa} = 57.0\ \text{MPa}$$

$$\sigma_b = 18.36\times10^{6} - 14.68\times10^{6} = 3.68\times10^{6}\ \text{Pa} = 3.68\ \text{MPa} \blacktriangleleft$$

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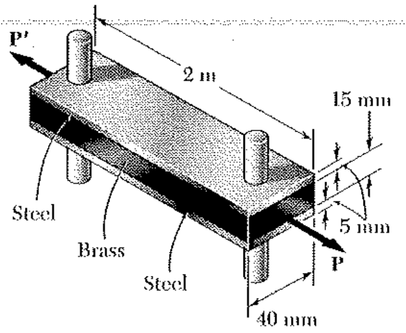
Problem 2.56



2.56 Determine the maximum load P that may be applied to the brass bar of Prob. 2.55 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

2.55 Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) which is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (a) the increase in temperature that was required to fit the steel bars on the pins, (b) the stress in the brass bar after the load is applied to it.

See solution to Problem 2.55 to obtain the fabrication stresses.

$\sigma_s^* = 22.03$ MPa $\qquad \sigma_b^* = -14.68$ MPa

Allowable stresses: $\sigma_{s,all} = 30$ MPa , $\sigma_{b,all} = 25$ MPa

Available stress increase from load.

$$\sigma_s = 30 - 22.03 = 7.97 \text{ MPa}$$
$$\sigma_b = 25 + 14.68 = 39.68 \text{ MPa}$$

Corresponding available strains.

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{7.97 \times 10^6}{200 \times 10^9} = 39.85 \times 10^{-6}$$
$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{39.68 \times 10^6}{105 \times 10^9} = 377.9 \times 10^{-6}$$

Smaller value governs $\therefore \quad \varepsilon = 39.85 \times 10^{-6}$

Areas: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$

$A_b = (15)(40) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$

$$P_s = E_s A_s \varepsilon = (200 \times 10^9)(400 \times 10^{-6})(39.85 \times 10^{-6}) = 3.188 \times 10^3 \text{ N}$$
$$P_b = E_b A_b \varepsilon = (105 \times 10^9)(600 \times 10^{-6})(39.85 \times 10^{-6}) = 2.511 \times 10^3 \text{ N}$$

Total allowable additional force.

$$P = P_s + P_b = 3.188 \times 10^3 + 2.511 \times 10^3 = 5.70 \times 10^3 \text{ N}$$
$$= 5.70 \text{ kN}$$

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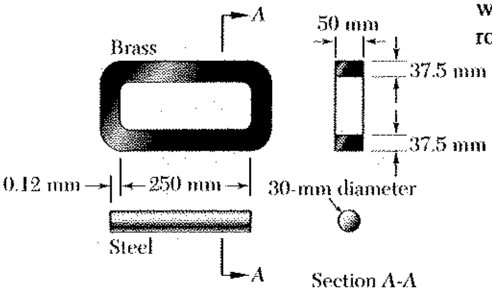
Problem 2.57

2.57 An brass link (E_b = 105 GPa, α_b= 20.9 × 10^{-6}/°C) and a steel rod (E_s = 200 GPa, α_s = 11.7 × 10^{-6}/°C) have the dimensions shown at a temperature of 20°C. The steel rod is cooled until it fits freely into the link. The temperature of the whole assembly is then raised to 45°C. Determine (*a*) the final stress in the steel rod, (*b*) the final length of the steel rod.



Initial dimensions at $T = 20°C$.

Final dimensions at $T = 45°C$.

$\Delta T = 45 - 20 = 25°C$

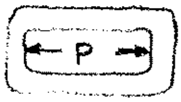
Free thermal expansion of each part.

Brass link: $(\delta_T)_b = \alpha_b(\Delta T)L' = (20.9\times10^{-6})(25)(0.250) = 130.625\times10^{-6}$ m

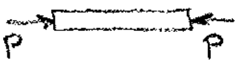
Steel rod: $(\delta_T)_s = \alpha_s(\Delta T)L = (11.7\times10^{-6})(25)(0.250) = 73.125\times10^{-6}$ m

At the final temperature the difference between the free length of the steel rod and the brass link is

$$\delta = 120\times10^{-6} + 73.125\times10^{-6} - 130.625\times10^{-6} = 62.5\times10^{-6}\ \text{m}$$



Add equal but opposite forces P to elongate the brass link and contract the steel rod.



Brass link: $E = 105\times10^9$ Pa

$A_b = (2)(50)(37.5) = 3750\ \text{mm}^2 = 3.750\times10^{-3}\ \text{m}^2$

$$(\delta_P) = \frac{PL}{EA} = \frac{P(0.250)}{(105\times10^9)(3.750\times10^{-3})} = 634.92\times10^{-12}P$$

Steel rod: $E = 200\times10^9$ Pa $\quad A_s = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^3 = 706.86\times10^{-6}\ \text{m}^2$

$$(\delta_P)_s = \frac{PL}{E_sA_s} = \frac{P(0.250)}{(200\times10^9)(706.86\times10^{-6})} = 1.76838\times10^{-9}P$$

$(\delta_P)_b + (\delta_P)_s = \delta \qquad 2.4033\times10^{-9}P = 62.5\times10^{-6}$

$P = 26.006\times10^3$ N

(a) Stress in steel rod. $\quad \sigma_s = -\frac{P}{A_s} = -\frac{(26.006\times10^3)}{706.86\times10^{-6}}$

$= -36.8\times10^6$ Pa $\qquad \sigma_s = -36.8$ MPa ◄

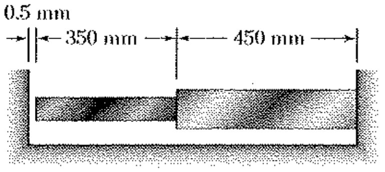
(b) Final length of steel rod. $\quad L_f = L_o + (\delta_T)_s - (\delta_P)_s$

$L_{f_1} = 0.250 + 120\times10^{-6} + 73.125\times10^{-6} - (1.76838\times10^{-9})(26.003\times10^3)$

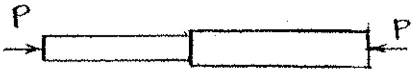
$= 0.250147$ m $\qquad L_f = 250.147$ mm ◄

Problem 2.58

2.58 Knowing that a 0.5-mm gap exists when the temperature is 24°C, determine (*a*) the temperature at which the normal stress in the aluminum bar will be equal to −75 MPa, (*b*) the corresponding exact length of the aluminum bar.



Bronze	Aluminum
$A = 1500\ \text{mm}^2$	$A = 1800\ \text{mm}^2$
$E = 105$ GPa	$E = 73$ GPa
$\alpha = 21.6 \times 10^{-6}/^\circ\text{C}$	$\alpha = 23.2 \times 10^{-6}/^\circ\text{C}$



$\sigma_a = -75$ MPa

$P = -\sigma_a A_a = -(75\times10^6)(1800\times10^{-6}) = -135$ kN

Shortening due to P

$$\delta_P = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a}$$
$$= \frac{(135000)(0.35)}{(105\times10^9)(1500\times10^{-6})} + \frac{(135000)(0.45)}{(73\times10^9)(1800\times10^{-6})}$$
$$= 762.3\times10^{-6}\ \text{m}.$$

Available elongation for thermal expansion

$$\delta_T = (0.0005 + 762.3\times10^{-6}) = 1.2623\times10^{-3}\ \text{m}$$

But $\delta_T = L_b\alpha_b(\Delta T) + L_a\alpha_a(\Delta T)$

$$= (0.35)(21.6\times10^{-6})(\Delta T) + (0.45)(23.2\times10^{-6})(\Delta T) = (18\times10^{-6})\Delta T$$

Equating $(18\times10^{-6})\Delta T = 1.2623\times10^{-3}$ $\qquad \Delta T = 70.1^\circ\text{C}$

(a) $T_{hot} = T_{cold} + \Delta T = 24 + 70.1 = 94.1^\circ\text{C}$

(b) $\delta_a = L_a\alpha_a(\Delta T) - \dfrac{PL_a}{E_a A_a}$

$$= (0.45)(23.2\times10^{-6})(70.1) - \frac{(135000)(0.45)}{(73\times10^9)(1800\times10^{-6})} = 0.27\times10^{-3}\ \text{m}.$$

$L_{exact} = 0.45 + 0.27\times10^{-3} = 0.45027$ m

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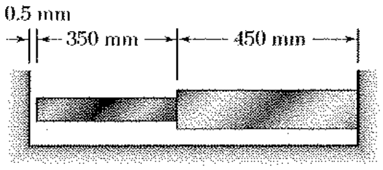
Problem 2.59

2.59 Determine (*a*) the compressive force in the bars shown after a temperature rise of 82°C, (*b*) the corresponding change in length of the bronze bar.



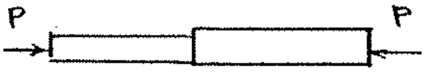
Bronze	Aluminum
$A = 1500\ \text{mm}^2$	$A = 1800\ \text{mm}^2$
$E = 105$ GPa	$E = 73$ GPa
$\alpha = 21.6 \times 10^{-6}/°\text{C}$	$\alpha = 23.2 \times 10^{-6}/°\text{C}$

Thermal expansion if free of constraint

$$\delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T)$$

$$= (0.35)(21.6\times10^{-6})(82) + (0.45)(23.2\times10^{-6})(82)$$

$$= 1.476 \times 10^{-3}\ m$$



Constrained expansion. $\delta = 0.5$ mm

Shortening due to induced compressive force P

$$\delta_P = 1.476\times10^{-3} - 0.0005 = 0.976\times10^{-3}\ m$$

But $\delta_P = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a} = \left(\frac{L_b}{E_b A_b} + \frac{L_a}{E_a A_a}\right)P$

$$= \left(\frac{0.35}{(105\times10^{9})(1500\times10^{-6})} + \frac{0.45}{(73\times10^{9})(1800\times10^{-6})}\right)P = 5.647\times10^{-9}.$$

(a) Equating $5.647\times10^{-9}P = 0.976\times10^{-3}$ $P = 172835$ N

172.8 kN ◄

(b) $\delta_b = L_b \alpha_b (\Delta T) - \frac{PL_b}{E_b A_b}$

$$= (0.35)(21.6\times10^{-3})(82) - \frac{(172835)(0.35)}{(105\times10^{9})(1500\times10^{-6})} = 0.2358\times10^{-3}\ m = 0.236\ mm$$ ◄

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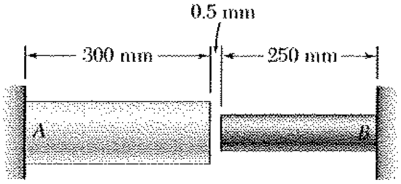
Problem 2.60

2.60 At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 140°C, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.



Aluminum	Stainless steel
$A = 2000\ \text{mm}^2$	$A = 800\ \text{mm}^2$
$E = 75\ \text{GPa}$	$E = 190\ \text{GPa}$
$\alpha = 23 \times 10^{-6}/°\text{C}$	$\alpha = 17.3 \times 10^{-6}/°\text{C}$

$$\Delta T = 140 - 20 = 120\ °\text{C}$$

Free thermal expansion.

$$\delta_T = L_a\alpha_a(\Delta T) + L_s\alpha_s(\Delta T)$$
$$= (0.300)(23\times10^{-6})(120) + (0.250)(17.3\times10^{-6})(120)$$
$$= 1.347\times10^{-3}\ \text{m}$$

Shortening due to P to meet constraint.

$$\delta_P = 1.347\times10^{-3} - 0.5\times10^{-3} = 0.847\times10^{-3}\ \text{m}$$

$$\delta_P = \frac{PL_a}{E_aA_a} + \frac{PL_s}{E_sA_s} = \left(\frac{L_a}{E_aA_a} + \frac{L_s}{E_sA_s}\right)P$$
$$= \left(\frac{0.300}{(75\times10^9)(2000\times10^{-6})} + \frac{0.250}{(190\times10^9)(800\times10^{-6})}\right)P = 3.6447\times10^{-9}\ P$$

Equating; $3.6447\times10^{-9}\ P = 0.847\times10^{-3}$ $\quad P = 232.39\times10^3\ \text{N}$

(a) $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{232.39\times10^3}{2000\times10^{-6}} = -116.2\times10^6\ \text{Pa}$ $\quad -116.2\ \text{MPa}$ ◄

(b) $\delta_a = L_a\alpha_a(\Delta T) - \dfrac{PL_a}{E_aA_a}$

$$= (0.300)(23\times10^{-6})(120) - \frac{(232.39\times10^3)(0.300)}{(75\times10^9)(2000\times10^{-6})} = 363\times10^{-6}\ \text{m}$$

0.363 mm ◄

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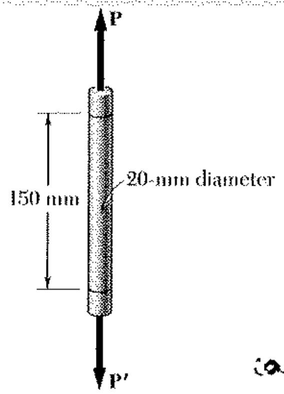
Problem 2.61

2.61 In a standard tensile test, an aluminum rod of 20-mm diameter is subjected to a tension force of $P = 30$ kN. Knowing that $\nu = 0.35$ and $E = 70$ GPa, determine (*a*) the elongation of the rod in an 150-mm gage length, (*b*) the change in diameter of the rod.



$P = 30 \times 10^6$ N

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(20)^2 = 314.159 \text{ mm}^2 = 314.159 \times 10^{-6} \text{ m}^2$

$\sigma_y = \frac{P}{A} = \frac{30 \times 10^3}{314.159 \times 10^{-6}} = 95.493 \times 10^6 \text{ Pa}$

$\varepsilon_y = \frac{\sigma_y}{E} = \frac{95.493 \times 10^6}{70 \times 10^6} = 1.36418 \times 10^{-3}$

(a) $\delta_y = L\varepsilon_y = (150 \times 10^{-3})(1.36418 \times 10^{-3}) = 205 \times 10^{-6}$ m

0.205 mm

$\varepsilon_x = -\nu\varepsilon_y = -(0.35)(1.36418 \times 10^{-3}) = -477.46 \times 10^{-6}$

(b) $\delta_x = d\,\varepsilon_x = (20 \times 10^{-3})(-477.46 \times 10^{-6}) = -9.55 \times 10^{-6}$ m

− 0.00955 mm

Problem 2.62

2.62 A 20-mm-diameter rod made of an experimental plastic is subjected to a tensile force of magnitude $P = 6$ kN. Knowing that an elongation of 14 mm and a decrease in diameter of 0.85 mm are observed in a 150-mm length, determine the modulus of elasticity, the modulus of rigidity and Poisson's ratio for the material.

Let the y-axis be along the length of the rod and the x-axis be transverse.

$A = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16 \times 10^{-6} \text{ m}^2$ $\qquad P = 6 \times 10^3$ N

$\sigma_y = \frac{P}{A} = \frac{6 \times 10^3}{314.16 \times 10^{-6}} = 19.0985 \times 10^6 \text{ Pa}$

$\varepsilon_y = \frac{\delta_y}{L} = \frac{14 \text{ mm}}{150 \text{ mm}} = 0.093333$

Modulus of elasticity: $E = \frac{\sigma_y}{\varepsilon_y} = \frac{19.0985 \times 10^6}{0.093333} = 204.63 \times 10^6 \text{ Pa}$

$E = 205$ MPa

$\varepsilon_x = \frac{\delta_x}{d} = -\frac{0.85}{20} = -0.0425$

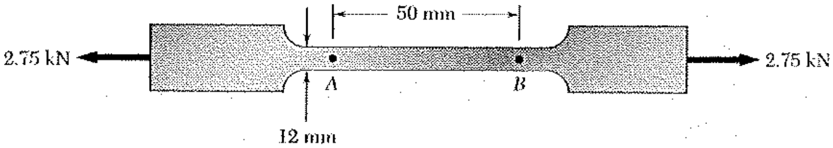
Poisson's ratio: $\nu = -\frac{\varepsilon_x}{\varepsilon_y} = -\frac{-0.0425}{0.093333}$ $\qquad \nu = 0.455$

Modulus of rigidity: $G = \frac{E}{2(1+\nu)} = \frac{204.63 \times 10^6}{(2)(1.455)} = 70.31 \times 10^6 \text{ Pa}$

$G = 70.3$ MPa

Problem 2.63

2.63 A 2.75 kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate ($E = 200$ GPa, $\nu = 0.30$). Determine the resulting change (*a*) in the 50-mm gage length, (*b*) in the width of portion *AB* of the test coupon, (*c*) in the thickness of portion *AB*, (*d*) in the cross-sectional area of portion *AB*.



$A = (1.6)(12) = 19.2 \text{ mm}^2$
$\quad = 19.2 \times 10^{-6} \text{ m}^2$

$P = 2.75 \times 10^3 \text{ N}$

$\sigma_x = \dfrac{P}{A} = \dfrac{2.75 \times 10^3}{19.2 \times 10^{-6}}$
$\quad = 143.229 \times 10^6 \text{ Pa}$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{143.229 \times 10^6}{200 \times 10^9} = 716.15 \times 10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -\nu \varepsilon_x = -(0.30)(716.15 \times 10^{-6}) = -214.84 \times 10^{-6}$$

(a) $L = 0.050 \text{ m}$ $\quad \delta_x = L\varepsilon_x = (0.050)(716.15 \times 10^{-6}) = 35.81 \times 10^{-6} \text{ m}$

0.0358 mm

(b) $w = 0.012 \text{ m}$ $\quad \delta_y = w\varepsilon_y = (0.012)(-214.84 \times 10^{-6}) = -2.578 \times 10^{-6} \text{ m}$

-0.00258 mm

(c) $t = 0.0016 \text{ m}$ $\quad \delta_z = t\varepsilon_z = (0.0016)(-214.84 \times 10^{-6}) = -3437 \times 10^{-9} \text{ m}$

-0.0003437 mm

(d) $A = w_0(1+\varepsilon_y)t_0(1+\varepsilon_z) = w_0 t_0 (1 + \varepsilon_y + \varepsilon_z + \varepsilon_y\varepsilon_z)$ $\qquad A_0 = w_0 t_0$

$\Delta A = A - A_0 = w_0 t_0 (\varepsilon_y + \varepsilon_z + \text{negligible term}) \approx 2 w_0 t_0 \varepsilon_y$

$\quad = (2)(0.012)(0.0016)(-214.84 \times 10^{-6}) = -8.25 \times 10^{-9} \text{ m}^2 = -0.00825 \text{ mm}^2$

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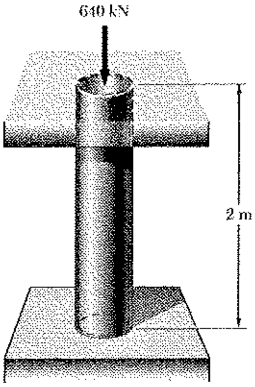
Probem 2.64

2.64 A 2-m length of an aluminum pipe of 240-mm outer diameter and 10-mm wall thickness is used as a short column and carries a centric axial load of 640 kN. Knowing that $E = 73$ GPa and $\nu = 0.33$, determine (*a*) the change in length of the pipe, (*b*) the change in its outer diameter, (*c*) the change in its wall thickness.



$d_o = 240$ mm $t = 10$ mm $d_i = d_o - 2t = 220$ mm

$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(240^2 - 220^2) = 7.2257\times10^3 \text{ mm}^2$

$= 7.2257\times10^{-3} \text{ m}^2$

$P = 640\times10^3$ N

(a) $\delta = -\frac{PL}{AE} = -\frac{(640\times10^3)(2.00)}{(7.2257\times10^{-3})(73\times10^9)} = -2.427\times10^{-3}$ m

$= -2.43$ mm ◄

$\varepsilon = \frac{\delta}{L} = -\frac{2.427\times10^{-3}}{2.00} = -1.2133\times10^{-3}$

$\varepsilon_{lat} = -\nu\varepsilon = -(0.33)(-1.2133\times10^{-3}) = 400.4\times10^{-6}$

(b) $\Delta d_o = d_o\,\varepsilon_{lat} = (240)(400.4\times10^{-6}) = 0.0961$ mm ◄

(c) $\Delta t = t\,\varepsilon_{lat} = (10)(400.4\times10^{-6}) = 0.00400$ mm ◄

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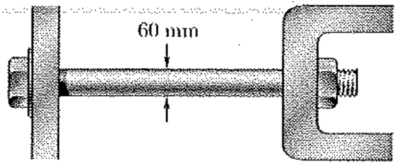
Problem 2.65



2.65 The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine the internal force in the bolt, if the diameter is observed to decrease by 13 μm.

$$\delta_y = -13\times10^{-6}\text{ m} \qquad d = 60\times10^{-3}\text{ m}$$

$$\varepsilon_y = \frac{\delta_y}{d} = -\frac{13\times10^{-6}}{60\times10^{-3}} = -216.67\times10^{-6}$$

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} \quad \therefore \quad \varepsilon_x = -\frac{\varepsilon_y}{\nu} = \frac{216.67\times10^{-6}}{0.29} = 747.13\times10^{-6}$$

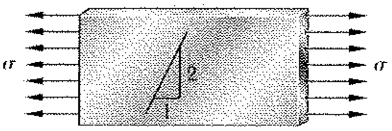
$$\sigma_x = E\varepsilon_y = (200\times10^{9})(747.13\times10^{-6}) = 149.43\times10^{6}\text{ Pa}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(60)^2 = 2.827\times10^{3}\text{ mm}^2 = 2.827\times10^{-3}\text{ m}^2$$

$$F = \sigma_x A = (149.43\times10^{6})(2.827\times10^{-3}) = 422\times10^{3}\text{ N}$$

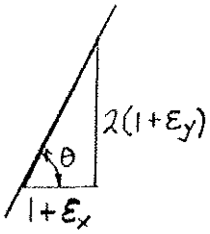
$$= 422\text{ kN}$$ ◄

Problem 2.66



2.66 An aluminum plate ($E = 74$ GPa and $\nu = 0.33$) is subjected to a centric axial load that causes a normal stress σ. Knowing that, before loading, a line of slope 2:1 is scribed on the plate, determine the slope of the line when $\sigma = 125$ MPa.

The slope after deformation is $\tan\theta = \dfrac{2(1+\varepsilon_y)}{1+\varepsilon_x}$



$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{125\times10^{6}}{74\times10^{9}} = 1.6892\times10^{-3}$$

$$\varepsilon_y = -\nu\varepsilon_x = -(0.33)(1.6892\times10^{-3}) = 0.5574\times10^{-3}$$

$$\tan\theta = \frac{2(1-0.0005574)}{1+0.0016892} = 1.99551$$ ◄

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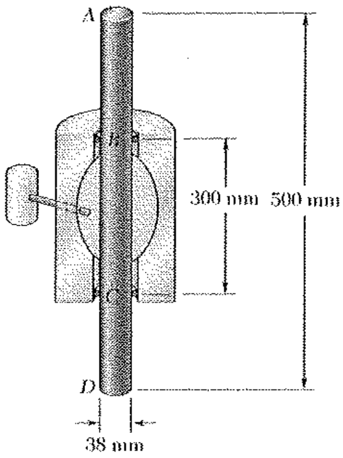
Problem 2.67



2.67 The aluminum rod *AD* is fitted with a jacket that is used to apply a hydrostatic pressure of 42 MPa to the 300-mm portion *BC* of the rod. Knowing that *E* = 70 GPa and *ν* = 0.36, determine (*a*) the change in the total length *AD*, (*b*) the change in diameter at the middle of the rod.

$\sigma_x = \sigma_z = -p = -42\,MPa \qquad \sigma_y = 0$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{70\times10^9}\left[-42\times10^6 - (0.36)(0) - (0.36)(-42\times10^6)\right]$$

$$= -384\times10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{70\times10^9}\left[-(0.36)(-42\times10^6) + 0 - (0.36)(-42\times10^6)\right]$$

$$= 432\times10^{-6}$$

Length subject to strain ε_x: $\qquad L = 0.3\,m$

(a) $\delta_y = L\varepsilon_y = (0.3)(432\times10^{-6}) = 0.1296\times10^{-3}\,m = 0.13\,mm$ ◄

(b) $\delta_x = d\varepsilon_x = (0.038)(-384\times10^{-6}) = -0.01459\times10^{-3}\,m = -0.0146\,mm$ ◄

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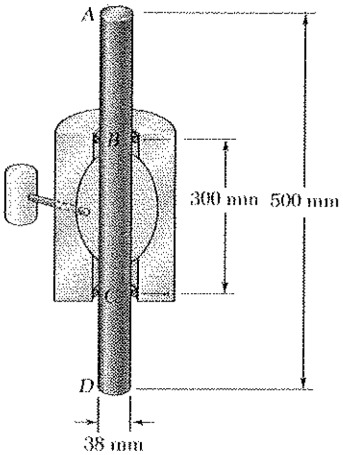
Problem 2.68



2.68 For the rod of Prob. 2.67, determine the forces that should be applied to the ends *A* and *D* of the rod (*a*) if the axial strain in portion *BC* of the rod is to remain zero as the hydrostatic pressure is applied, (*b*) if the total length *AD* of the rod is to remain unchanged.

2.67 The aluminum rod *AD* is fitted with a jacket that is used to apply a hydrostatic pressure of 42 MPa to the 300-mm portion *BC* of the rod. Knowing that $E = 70$ GPa and $\nu = 0.36$, determine (*a*) the change in the total length *AD*, (*b*) the change in diameter at the middle of the rod.

Over the pressurized portion BC

$$\sigma_x = \sigma_z = -p \qquad \sigma_y = \sigma_y$$

$$(\varepsilon_y)_{BC} = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{E}(2\nu p + \sigma_y)$$

(a) $(\varepsilon_y)_{BC} = 0 \qquad 2\nu p + \sigma_y = 0$

$$\sigma_y = 2\nu p = -(2)(0.36)(42\times10^6)$$

$$= -30.24 \text{ MPa}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.038)^2 = 1.134\times10^{-3} \text{ m}^2$$

$$F = A\sigma_y = (1.134\times10^{-3})(-30.24\times10^6) = -34292 \text{ N}$$

i.e. 34.3 kN compression ◀

(b) Over unpressurized portions AB and CD $\quad \sigma_x = \sigma_z = 0$

$$(\varepsilon_y)_{AB} = (\varepsilon_y)_{CD} = \frac{\sigma_y}{E}$$

For no change in length

$$\delta = L_{AB}(\varepsilon_y)_{AB} + L_{BC}(\varepsilon_y)_{BC} + L_{CD}(\varepsilon_y)_{CD} = 0$$

$$(L_{AB} + L_{CD})(\varepsilon_y)_{AB} + L_{BC}(\varepsilon_y)_{BC} = 0$$

$$(0.5 - 0.3)\frac{\sigma_y}{E} + \frac{0.3}{E}(2\nu p + \sigma_y) = 0$$

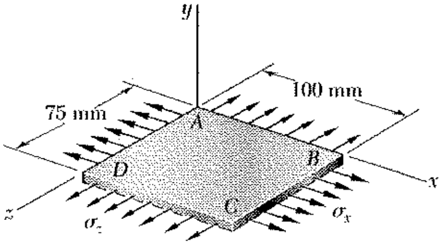
$$\sigma_y = -\frac{0.6\nu p}{0.1} = -\frac{(0.6)(0.36)(42\times10^6)}{0.1} = -90.72 \text{ MPa}$$

$$P = A\sigma_y = (1.134\times10^{-3})(-90.72\times10^6) = -102876 \text{ N}$$

i.e 102.9 kN compression ◀

Problem 2.69

2.69 A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 120$ MPa and $\sigma_z = 160$ MPa. Knowing that the properties of the fabric can be approximated as $E = 87$ GPa and $\nu = 0.34$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.



$$\sigma_x = 120\times10^{6}\ \text{Pa},\quad \sigma_y = 0,\quad \sigma_z = 160\times10^{6}\ \text{Pa}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{87\times10^{9}}\left[120\times10^{6} - (0.34)(160\times10^{6})\right]$$

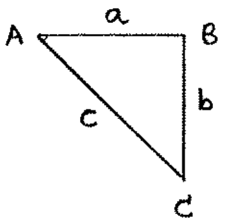
$$= 754.02\times10^{-6}$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{87\times10^{9}}\left[-(0.34)(120\times10^{6}) + 160\times10^{6}\right]$$

$$= 1.3701\times10^{-3}$$

(a) $\delta_{AB} = (\overline{AB})\varepsilon_x = (100\ \text{mm})(754.02\times10^{-6}) = 0.0754\ \text{mm}$ ◀

(b) $\delta_{BC} = (\overline{BC})\varepsilon_z = (.75\ \text{mm})(1.3701\times10^{-6}) = 0.1028\ \text{mm}$ ◀



Label sides of right triangle ABC as a, b, and c.

$$c^2 = a^2 + b^2$$

Obtain differentials by calculus.

$$2c\,dc = 2a\,da + 2b\,db$$

$$dc = \frac{a}{c}\,da + \frac{b}{c}\,db$$

But, $a = 100$ mm, $b = 100$ mm, $c = \sqrt{100^2 + .75^2)} = 125$ mm

$da = \delta_{AB} = 0.0754$ mm $\qquad db = \delta_{BC} = 0.1370$ mm

(c) $\delta_{AC} = dc = \frac{100}{125}(0.0754) + \frac{75}{125}(0.1028) = 0.1220\ \text{mm}$ ◀

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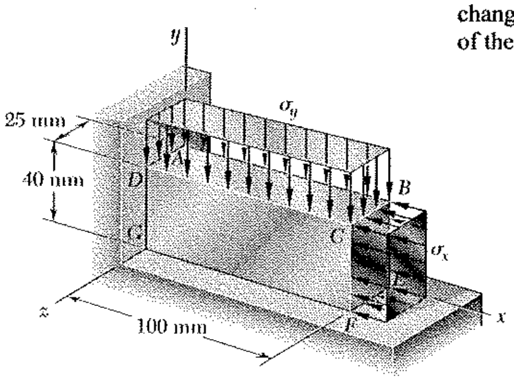
Problem 2.70

2.70 The block shown is made of a magnesium alloy for which $E = 45$ GPa and $\nu = 0.35$. Knowing that $\sigma_x = -180$ MPa, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.



$\delta_y = 0 \qquad \varepsilon_y = 0 \qquad \sigma_z = 0$

$\varepsilon_y = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$

$\sigma_y = \nu\sigma_x = (0.35)(-180\times10^6)$

$= -63\times10^6$ Pa $\qquad \sigma_y = -63$ MPa ◄

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{(0.35)(-243\times10^6)}{45\times10^9} = -1.89\times10^{-3}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x - \nu\sigma_y}{E} = -\frac{157.95\times10^6}{45\times10^9} = -3.51\times10^{-3}$$

(b) $A_o = L_x L_z$

$A = L_x(1+\varepsilon_x)L_z(1+\varepsilon_z) = L_x L_z(1 + \varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z)$

$\Delta A = A - A_o = L_x L_z(\varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z) \approx L_x L_z(\varepsilon_x + \varepsilon_z)$

$\Delta A = (100\text{ mm})(25\text{ mm})(-3.51\times10^{-3} - 1.89\times10^{-3}) \qquad \Delta A = -13.50\text{ mm}^2$ ◄

(c) $\mathcal{V}_o = L_x L_y L_z$

$\mathcal{V} = L_x(1+\varepsilon_x)L_y(1+\varepsilon_y)L_z(1+\varepsilon_z)$

$= L_x L_y L_z(1 + \varepsilon_x + \varepsilon_y + \varepsilon_z + \varepsilon_x\varepsilon_y + \varepsilon_y\varepsilon_z + \varepsilon_z\varepsilon_x + \varepsilon_x\varepsilon_y\varepsilon_z)$

$\Delta\mathcal{V} = \mathcal{V} - \mathcal{V}_o = L_x L_y L_z(\varepsilon_x + \varepsilon_y + \varepsilon_z + \text{small terms})$

$\Delta\mathcal{V} = (100)(40)(25)(-3.51\times10^{-3} + 0 - 1.89\times10^{-3}) \qquad \Delta\mathcal{V} = -540\text{ mm}^3$ ◄

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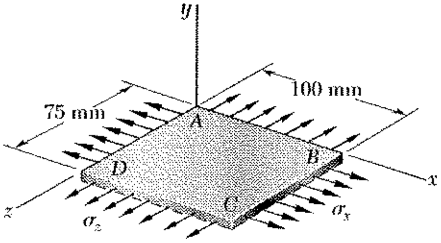
Problem 2.71



2.71 The homogeneous plate *ABCD* is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the x direction must be zero, that is, $\varepsilon_x = 0$. Denoting by E the modulus of elasticity and by ν Poisson's ratio, determine (*a*) the required magnitude of σ_x, (*b*) the ratio σ_0 / ε_z.

$$\sigma_z = \sigma_0 , \quad \sigma_y = 0 , \quad \varepsilon_x = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}(\sigma_x - \nu\sigma_0)$$

(a) $\sigma_x = \nu\sigma_0$

(b) $\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{E}(-\nu^2\sigma_0 - 0 + \sigma_0) = \frac{1-\nu^2}{E}\sigma_0$

$$\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-\nu^2}$$

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Problem 2.72

2.72 For a member under axial loading, express the normal strain ε' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ε_x by (*a*) comparing the hypothenuses of the triangles shown in Fig. 2.54, which represent respectively an element before and after deformation, (*b*) using the values of the corresponding stresses σ' and σ_x shown in Fig. 1.40, and the generalized Hooke's law.

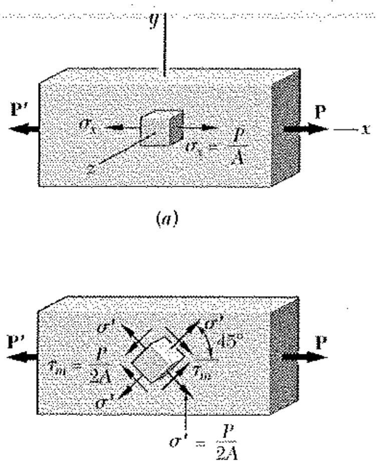


Fig. 1.40 (*b*)

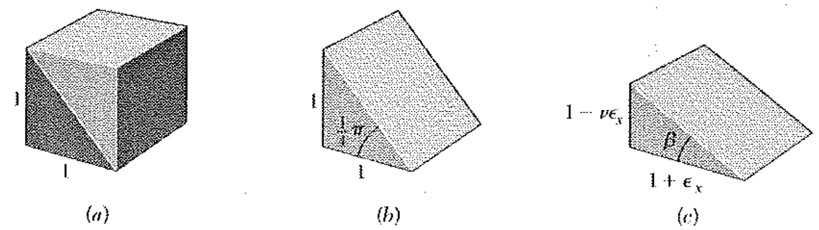
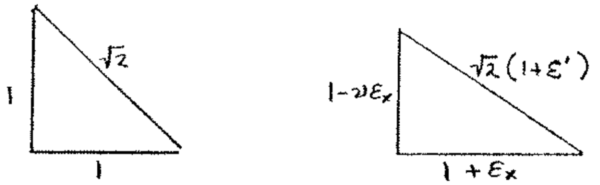


Fig. 2.54



(a) Before deformation After deformation

$$\left[\sqrt{2}(1+\varepsilon')\right]^2 = (1+\varepsilon_x)^2 + (1-\nu\varepsilon_x)^2$$

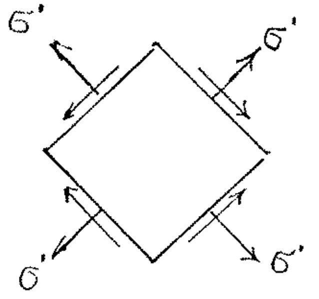
$$2(1+2\varepsilon'+\varepsilon'^2) = 1 + 2\varepsilon_x + \varepsilon_x^2 + 1 - 2\nu\varepsilon_x + \nu^2\varepsilon_x^2$$

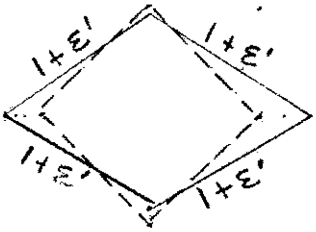
$$4\varepsilon' + 2\varepsilon'^2 = 2\varepsilon_x + \varepsilon_x^2 - 2\nu\varepsilon_x + \nu^2\varepsilon_x^2$$

Neglect squares as small $\quad 4\varepsilon' = 2\varepsilon_x - 2\nu\varepsilon_x$

$$\varepsilon' = \frac{1-\nu}{2}\varepsilon_x$$

(b)





$$\varepsilon' = \frac{\sigma'}{E} - \frac{\nu\sigma'}{E}$$

$$= \frac{1-\nu}{E}\cdot\frac{P}{2A}$$

$$= \frac{1-\nu}{2E}\sigma_x$$

$$= \frac{1-\nu}{2}\varepsilon_x$$

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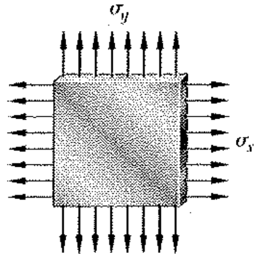
Problem 2.73



2.73 In many situations it is known that the normal stress in a given direction is zero, for example $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress*, show that if the strains ε_x and ε_y have been determined experimentally, we can express σ_x, σ_y and ε_z as follows:

$$\sigma_x = E\frac{\varepsilon_x + \nu\varepsilon_y}{1-\nu^2} \qquad \sigma_y = E\frac{\varepsilon_y + \nu\varepsilon_x}{1-\nu^2} \qquad \varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

$$\sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \quad (1) \qquad \varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y) \quad (2)$$

Multiplying (2) by ν and adding to (1)

$$\varepsilon_x + \nu\varepsilon_y = \frac{1-\nu^2}{E}\sigma_x \qquad \text{or} \qquad \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \quad ◄$$

Multiplying (1) by ν and adding to (2)

$$\varepsilon_y + \nu\varepsilon_x = \frac{1-\nu^2}{E}\sigma_y \qquad \text{or} \qquad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) \quad ◄$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{\cancel{E}}\cdot\frac{\cancel{E}}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y + \varepsilon_y + \nu\varepsilon_x)$$

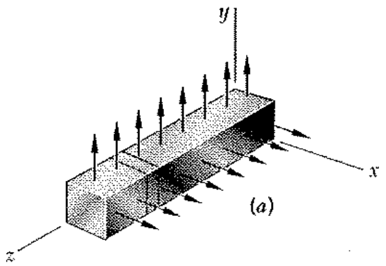
$$= -\frac{\nu(1+\nu)}{1-\nu^2}(\varepsilon_x + \varepsilon_y) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) \quad ◄$$

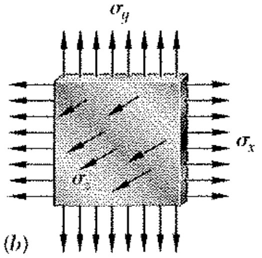
Problem 2.74

2.74 In many situations physical constraints prevent strain from occurring in a given direction, for example $\varepsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation, which is known as *plane strain*, we can express σ_z, ε_x and ε_y as follows:

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y] \qquad \varepsilon_y = \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$





$$\varepsilon_z = 0 = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) \qquad \text{or} \qquad \sigma_z = \nu(\sigma_x + \sigma_y) \quad ◄$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}\left[\sigma_x - \nu\sigma_y - \nu^2(\sigma_x + \sigma_y)\right]$$

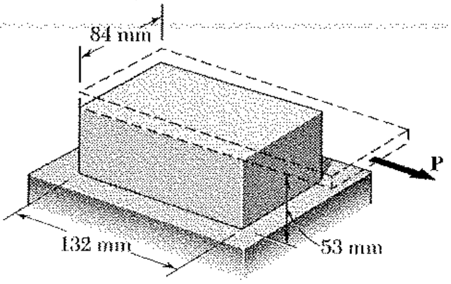
$$= \frac{1}{E}\left[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y\right] \quad ◄$$

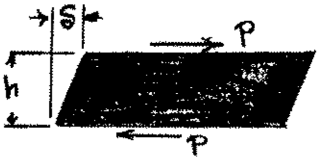
$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1}{E}\left[-\nu\sigma_x + \sigma_y - \nu^2(\sigma_x + \sigma_y)\right]$$

$$= \frac{1}{E}\left[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x\right] \quad ◄$$

Problem 2.75

2.75 The plastic block shown is bonded to a fixed base and to a horizontal rigid plate to which a force **P** is applied. Knowing that for the plastic used $G = 385$ MPa, determine the deflection of the plate when $P = 36$ kN.





Consider the plastic block. The shearing force carried is P = 36 kN

The area is $A = (0.084)(0.132) = 11.088 \times 10^{-3}\ m^2$

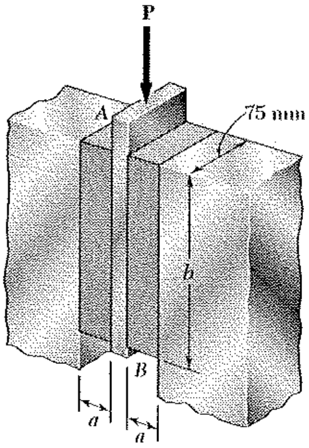
Shearing stress. $\tau = \frac{P}{A} = \frac{36 \times 10^3}{11.088 \times 10^{-3}} = 3.24675$ MPa.

Shearing strain. $\gamma = \frac{\tau}{G} = \frac{3.24675}{385} = 0.008433$

But $\gamma = \frac{\delta}{h}$ ∴ $\delta = h\gamma = (53)(0.008433) = 0.45$ mm ◀

Problem 2.76

2.76 A vibration isolation unit consists of two blocks of hard rubber bonded to plate *AB* and to rigid supports as shown. For the type and grade of rubber used $\tau_{all} = 1.54$ MPa and $G = 12.6$ MPa. Knowing that a centric vertical force of magnitude $P = 13$ kN must cause a 2.5-mm vertical deflection of the plate *AB*, determine the smallest allowable dimensions *a* and *b* of the block.



Consider the rubber block on the right. It carries a shearing force equal to ½P

The shearing stress is $\tau = \frac{\frac{1}{2}P}{A}$

or required $A = \frac{P}{2\tau} = \frac{13 \times 10^3}{(2)(1.54 \times 10^6)} = 4.221 \times 10^{-3}\ m^2$

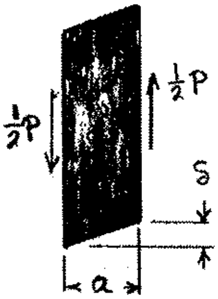
But $A = (3.0)\ b$

Hence $b = \frac{A}{0.075} = 0.05628$ m $b_{min} = 56.3$ mm ◀

Use $b = 56.3$ mm and $\tau = 1.54$ MPa

Shearing strain. $\gamma = \frac{\tau}{G} = \frac{1.54}{12.6} = 0.12222$

But, $\gamma = \frac{\delta}{a}$

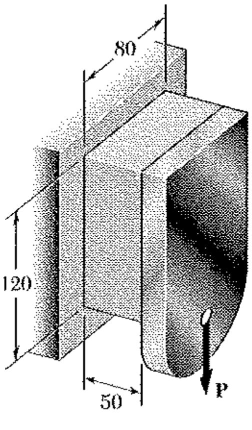


Hence, $a = \frac{\delta}{\gamma} = \frac{2.5}{0.12222} = 20.45$ mm

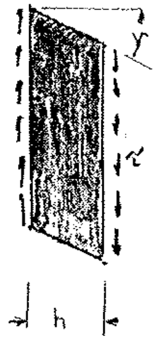
$a_{min} = 20.5$ mm ◀

Problem 2.77

2.77 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 240-kN load **P** is applied. Knowing that for the plastic used G = 1050 MPa, determine the deflection of the plate.



Dimensions in mm



$A = (80)(120) = 9.6 \times 10^3\ \text{mm}^2 = 9.6 \times 10^{-3}\ \text{m}^2$

$P = 240 \times 10^3\ \text{N}$

$\tau = \frac{P}{A} = \frac{240 \times 10^3}{9.6 \times 10^{-3}} = 25 \times 10^6\ \text{Pa}$

$G = 1050 \times 10^6\ \text{Pa}$

$\gamma = \frac{\tau}{G} = \frac{25 \times 10^6}{1050 \times 10^6} = 23.810 \times 10^{-3}$

$h = 50\ \text{mm} = 0.050\ \text{m}$

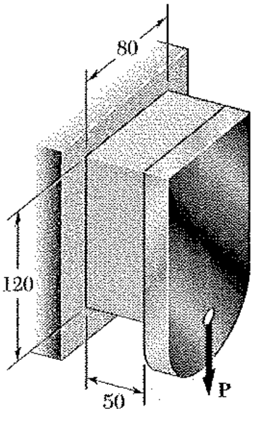
$\delta = h\gamma = (0.050)(23.810 \times 10^{-3})$

$= 1.190 \times 10^{-3}\ \text{m}$ 1.091 mm ↓ ◀

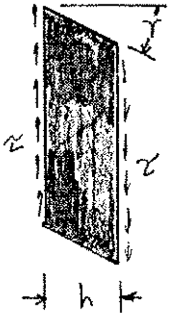
Problem 2.78

2.78 What load **P** should be applied to the plate of Prob. 2.77 to produce a 1.5-mm deflection?

2.77 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 240-kN load **P** is applied. Knowing that for the plastic used G = 1050 MPa, determine the deflection of the plate.



Dimensions in mm



$\delta = 1.5\ \text{mm} = 1.5 \times 10^{-3}\ \text{m}$

$h = 50\ \text{mm} = 50 \times 10^{-3}\ \text{m}$

$\gamma = \frac{\delta}{h} = \frac{1.5 \times 10^{-3}}{50 \times 10^{-3}} = 30 \times 10^{-3}$

$G = 1050 \times 10^6\ \text{Pa}$

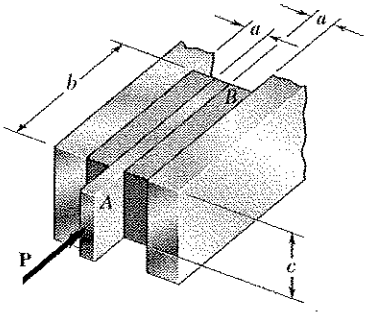
$\tau = G\gamma = (1050 \times 10^6)(30 \times 10^{-3})$
$= 31.5 \times 10^6\ \text{Pa}$

$A = (120)(80) = 9.6 \times 10^3\ \text{mm}^2 = 9.6 \times 10^{-3}\ \text{m}^2$

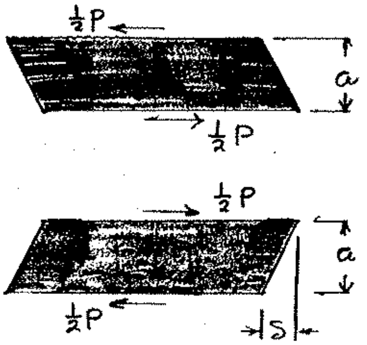
$P = \tau A = (31.5 \times 10^6)(9.6 \times 10^{-3}) = 302 \times 10^3\ \text{N}$ 302 kN ◀

NOTE: In problems 2.75 through 2.82, 2.87, 2.88, and 2.132 it is assumed that γ is the tangent of the angle change. This makes the relative displacement proportional to the force.

Problem 2.79



2.79 Two blocks of rubber with a modulus of rigidity $G = 12$ MPa are bonded to rigid supports and to a plate AB. Knowing that $c = 100$ mm and $P = 40$ kN, determine the smallest allowable dimensions a and b of the blocks if the shearing stress in the rubber is not to exceed 1.4 MPa and the deflection of the plate is to be at least 5 mm.



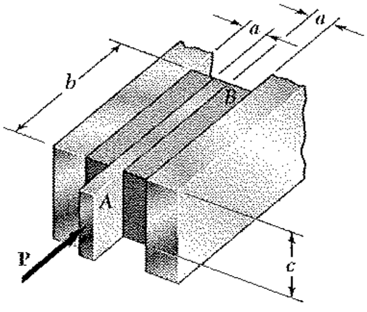
Shearing strain $\gamma = \frac{\delta}{a} = \frac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(12\times10^6)(0.005)}{1.4\times10^6} = 0.04286 \text{ m} = 43 \text{ mm.}$$

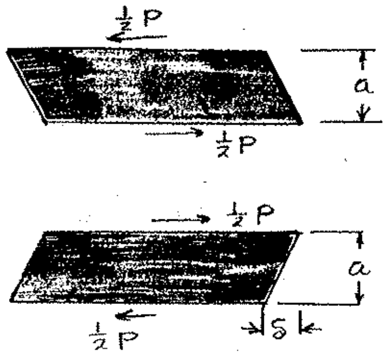
Shearing stress $\tau = \frac{\frac{1}{2}P}{A} = \frac{P}{2bc}$

$$b = \frac{P}{2c\tau} = \frac{40\times10^3}{(2)(0.1)(1.4\times10^6)} = 0.143 \text{ m} = 143 \text{ mm.}$$

Problem 2.80



2.80 Two blocks of rubber with a modulus of rigidity $G = 10$ MPa are bonded to rigid supports and to a plate AB. Knowing that $b = 200$ mm and $c = 125$ mm, determine the largest allowable load P and the smallest allowable thickness a of the blocks if the shearing stress in the rubber is not to exceed 1.5 MPa and the deflection of the plate is to be at least 6 mm.



Shearing stress $\tau = \frac{\frac{1}{2}P}{A} = \frac{P}{2bc}$

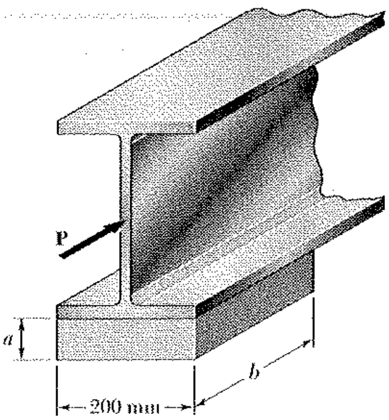
$$P = 2bc\tau = (2)(0.2)(0.125)(1.5\times10^6) = 75 \text{ kN}$$

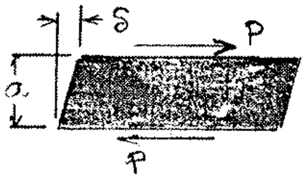
Shearing strain $\gamma = \frac{\delta}{a} = \frac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(10\times10^6)(0.006)}{1.5\times10^6} = 0.04 \text{ m} = 40 \text{ mm}$$

Problem 2.81

2.81 An elastomeric bearing ($G = 0.9$ MPa) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22-kN lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 420 kPa, determine (*a*) the smallest allowable dimension *b*, (*b*) the smallest required thickness *a*.





Shearing force $P = 22 \times 10^3$ N

Shearing stress $\tau = 420 \times 10^3$ Pa

$$\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau} = \frac{22 \times 10^3}{420 \times 10^3} = 52.381 \times 10^{-3}\ \text{m}^2$$

$$= 52.381 \times 10^3\ \text{mm}^2$$

$$A = (200\ \text{mm})(b)$$

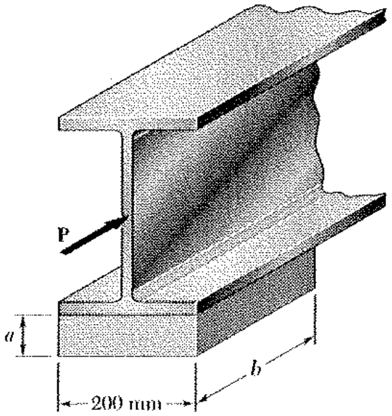
(a) $$b = \frac{A}{200} = \frac{52.381 \times 10^3}{200} = 262\ \text{mm}$$ ◄

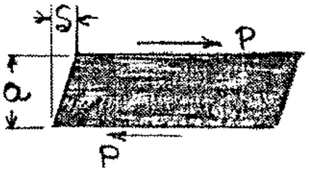
$$\gamma = \frac{\tau}{G} = \frac{420 \times 10^3}{0.9 \times 10^6} = 466.67 \times 10^{-3}$$

(b) But $\gamma = \frac{\delta}{a}$ $\therefore$ $$a = \frac{\delta}{\gamma} = \frac{10\ \text{mm}}{466.67 \times 10^{-3}} = 21.4\ \text{mm}$$ ◄

Problem 2.82

2.82 For the elastomeric bearing in Prob. 2.81 with $b = 220$ mm and $a = 30$ mm, determine the shearing modulus G and the shear stress τ for a maximum lateral load $P = 19$ kN and a maximum displacement $\delta = 12$ mm.





Shearing force $P = 19 \times 10^3$ N

Area $A = (200\ \text{mm})(220\ \text{mm}) = 44 \times 10^3\ \text{mm}^2$
$= 44 \times 10^{-3}\ \text{m}^2$

$$\tau = \frac{P}{A} = \frac{19 \times 10^3}{44 \times 10^{-3}} = 431.81 \times 10^3\ \text{Pa}$$
$$= 431\ \text{kPa}$$ ◄

Shearing strain $$\gamma = \frac{\delta}{a} = \frac{12\ \text{mm}}{30\ \text{mm}} = 0.400$$

Shearing modulus

$$G = \frac{\tau}{\gamma} = \frac{431.81 \times 10^3}{0.4} = 1.080 \times 10^6\ \text{Pa}$$
$$= 1.080\ \text{MPa}$$ ◄

NOTE: In problems 2.75 through 2.82, 2.87, 2.88, and 2.132 it is assumed that γ is the tangent of the angle change. This makes the relative displacement proportional to the force.

Problem 2.83

***2.83** A 150-mm diameter solid steel sphere is lowered into the ocean to a point where the pressure is 50 MPa (about 5 km below the surface). Knowing that $E = 200$ GPa and $\nu = 0.30$, determine (*a*) the decrease in diameter of the sphere, (*b*) the decrease in volume of the sphere, (*c*) the percent increase in the density of the sphere.

For a solid sphere $\mathcal{V}_o = \frac{\pi}{6} d_o^3 = \frac{\pi}{6}(0.15)^3 = 1.767 \times 10^{-3}\ m^3.$

$$\sigma_x = \sigma_y = \sigma_z = -p = 50 MPa.$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = -\frac{(1-2\nu)p}{E \;\; -6} = -\frac{(0.4)(50\times10^6)}{200\times10^9} = -100\times10^{-6}$$

Likewise $\varepsilon_y = \varepsilon_z = -100 \times 10$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -300\times10^{-6}$$

(a) $-\Delta d = -d_o\varepsilon_x = -(0.15)(100\times10^{-6}) = 15\times10^{-6} m.$ ◄

(b) $-\Delta\mathcal{V} = -\mathcal{V}_o e = -(1.767\times10^{-3})(-300\times10^{-6}) = 530\times10^{-9}\ m^3$ ◄

(c) Let m = mass of sphere, m = constant.

$$m = \rho_o\mathcal{V}_o = \rho\mathcal{V} = \rho\mathcal{V}_o(1+e)$$

$$\frac{\rho-\rho_o}{\rho_o} = \frac{\rho}{\rho_o} - 1 = \frac{m}{\mathcal{V}_o(1+e)}\cdot\frac{\mathcal{V}_o}{m} - 1 = \frac{1}{1+e} - 1$$

$$= (1 - e + e^2 - e^3 + \dots) - 1 = -e + e^2 - e^3 + \dots$$

$$\approx -e = 300\times10^{-6}$$

$\frac{\rho-\rho_o}{\rho_o}\times 100\% = (300\times10^{-6})(100\%) = 0.03\%$ ◄

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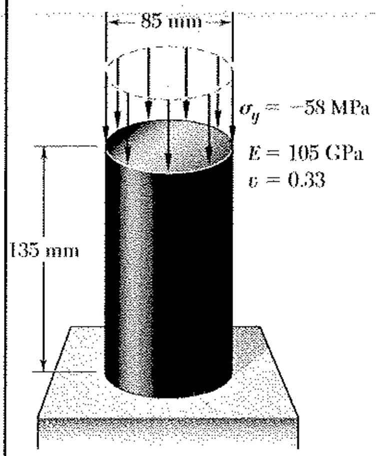
Problem 2.84

***2.84** (*a*) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (*b*) Solve part *a*, assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70$ MPa.



$h_o = 135 \text{ mm} = 0.135 \text{ m}$

$A_o = \frac{\pi}{4} d_o^2 = \frac{\pi}{4}(85)^2 = 5.6745 \times 10^3 \text{ mm}^2 = 5.6745 \times 10^{-3} \text{ m}^2$

$\mathcal{V}_o = A_o h_o = 766.06 \times 10^3 \text{ mm}^3 = 766.06 \times 10^{-6} \text{ m}^3$

(a) $\sigma_x = 0$, $\sigma_y = -58 \times 10^6$ Pa, $\sigma_z = 0$

$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{\sigma_y}{E}$

$= -\frac{58 \times 10^6}{105 \times 10^9} = -552.38 \times 10^{-6}$

$\Delta h = h_o \varepsilon_y = (135 \text{ mm})(-552.38 \times 10^{-6}) = -0.0746 \text{ mm}$

$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(1-2\nu)\sigma_y}{E} = \frac{(0.34)(-58 \times 10^6)}{105 \times 10^9}$

$= -187.81 \times 10^{-6}$

$\Delta \mathcal{V} = \mathcal{V}_o e = (766.06 \times 10^3 \text{ mm}^3)(-187.81 \times 10^{-6}) = -143.9 \text{ mm}^3$

(b) $\sigma_x = \sigma_y = \sigma_z = -70 \times 10^6$ Pa $\qquad \sigma_x + \sigma_y + \sigma_z = -210 \times 10^6$ Pa

$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1-2\nu}{E}\sigma_y$

$= \frac{(0.34)(-70 \times 10^6)}{105 \times 10^9} = -226.67 \times 10^{-6}$

$\Delta h = h_o \varepsilon_y = (135 \text{ mm})(-226.67 \times 10^{-6}) = -0.0306 \text{ mm}$

$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(0.34)(-210 \times 10^6)}{105 \times 10^9} = -680 \times 10^{-6}$

$\Delta \mathcal{V} = \mathcal{V}_o e = (766.06 \times 10^3 \text{ mm}^3)(-680 \times 10^{-6}) = -521 \text{ mm}^3$

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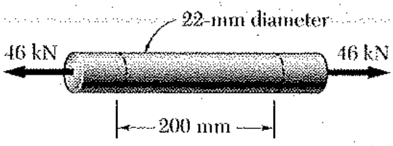
Problem 2.85

2.85 Determine the dilatation *e* and the change in volume of the 200-mm length of the rod shown if (*a*) the rod is made of steel with *E* = 200 GPa and *ν* = 0.30, (*b*) the rod is made of aluminum with *E* = 70 GPa and *ν* = 0.35.



$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(22)^2 = 380.13\ \text{mm}^2 = 380.13 \times 10^{-6}\ \text{m}^2$$

$P = 46 \times 10^3\ \text{N}$ $\qquad \sigma_x = \frac{P}{A} = 121.01 \times 10^6\ \text{Pa}$ $\qquad \sigma_y = \sigma_z = 0$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x}{E} \qquad \varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -\nu\frac{\sigma_x}{E}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{E}(\sigma_x - \nu\sigma_x - \nu\sigma_x) = \frac{(1-2\nu)\sigma_x}{E}$$

Volume $\mathcal{V} = AL = (380.13\ \text{mm}^2)(200\ \text{mm}) = 76.026 \times 10^3\ \text{mm}^3$

$$\Delta\mathcal{V} = \mathcal{V}e$$

(a) steel: $e = \frac{(0.4)(121.01 \times 10^6)}{200 \times 10^9} = 242 \times 10^{-6}$ ◀

$\Delta\mathcal{V} = (76.026 \times 10^3)(242 \times 10^{-6}) = 18.40\ \text{mm}^3$ ◀

(b) aluminum: $e = \frac{(0.3)(121.01 \times 10^6)}{70 \times 10^9} = 519 \times 10^{-6}$ ◀

$\Delta\mathcal{V} = (76.026 \times 10^3)(519 \times 10^{-6}) = 39.4\ \text{mm}^3$ ◀

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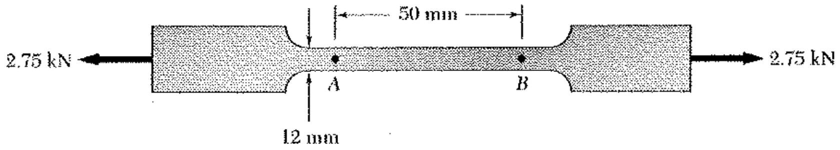
Problem 2.86

*2.86 Determine the change in volume of the 50-mm gage length segment *AB* in Prob. 2.63 (*a*) by computing the dilatation of the material, (*b*) by subtracting the original volume of portion *AB* from its final volume.

2.63 A 2.75 kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate ($E = 200$ GPa, $\nu = 0.30$). Determine the resulting change (*a*) in the 50-mm gage length, (*b*) in the width of portion *AB* of the test coupon, (*c*) in the thickness of portion *AB*, (*d*) in the cross-sectional area of portion *AB*.

All calculations here are in mm



(a) $A_o = (12)(1.6) = 19.2 \text{ mm}^2 = 19.2 \times 10^{-6} \text{ m}^2$

Volume $\quad \mathcal{V}_o = L_o A_o = (50)(19.2) = 960 \text{ mm}^3$

$$\sigma_x = \frac{P}{A_o} = \frac{2.75 \times 10^3}{19.2 \times 10^{-6}} = 143.229 \times 10^6 \text{ Pa} \qquad \sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x}{E} = \frac{143.229 \times 10^6}{200 \times 10^9} = 716.15 \times 10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -(0.30)(716.15 \times 10^{-3}) = -214.84 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 286.46 \times 10^{-6}$$

$$\Delta\mathcal{V} = \mathcal{V}_o e = (960)(286.46 \times 10^{-6}) = 0.275 \text{ mm}^3$$ ◄

(b) From the solution to Problem 2.62

$\delta_x = 0.03581$ mm $\qquad \delta_y = -0.002578$ mm $\qquad \delta_z = -0.0003437$ mm

The dimensions when under the 2.75 kN load are:

length $\quad L = L_o + \delta_x = 50 + 0.03581 \quad 50.03581$ mm

width $\quad w = w_o + \delta_y = 12 - 0.002578 = 11.997422$ mm

thickness $\quad t = t_o + \delta_z = 1.6 - 0.0003437 = 1.5996563$ mm

volume $\quad \mathcal{V} = Lwt = (50.03581)(11.997422)(1.5996563) = 960.275 \text{ mm}^3$

$$\Delta\mathcal{V} = \mathcal{V} - \mathcal{V}_o = 960.275 - 960 = 0.275 \text{ mm}^3$$ ◄

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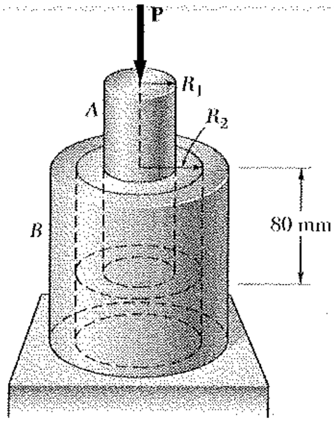
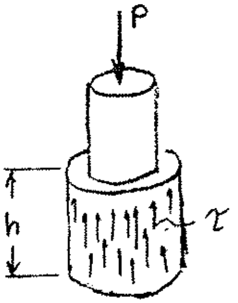
Problem 2.87

***2.87** A vibration isolation support consists of a rod *A* of radius $R_1 = 10$ mm and a tube *B* of inner radius $R_2 = 25$ mm bonded to an 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 12$ MPa. Determine the largest allowable force **P** which can be applied to rod *A* if its deflection is not to exceed 2.50 mm.



Let r be a radial coordinate. Over the hollow rubber cylinder $R_1 \leq r \leq R_2$

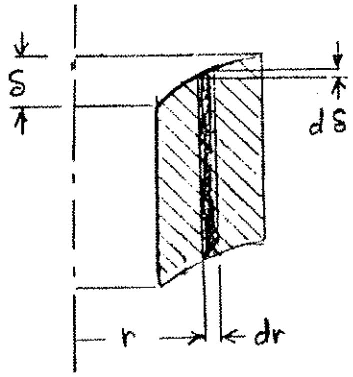


Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$



Shearing deformation over radial length dr

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi G h}\frac{dr}{r}$$

Total deformation

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi G h}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi G h}\ln r \Big|_{R_1}^{R_2} = \frac{P}{2\pi G h}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi G h}\ln\frac{R_2}{R_1} \quad \text{or} \quad P = \frac{2\pi G h \delta}{\ln(R_2/R_1)}$$

Data: $R_1 = 10\text{ mm} = 0.010\text{ m}$, $R_2 = 25\text{ mm} = 0.025\text{ m}$, $h = 80\text{ mm} = 0.080\text{ m}$

$G = 12\times10^6\text{ Pa}$ $\qquad \delta = 2.50\times10^{-3}\text{ m}$

$$P = \frac{(2\pi)(12\times10^6)(0.080)(2.50\times10^{-3})}{\ln(0.025/0.010)} = 16.46\times10^3\text{ N} \qquad 16.46\text{ kN} \blacktriangleleft$$

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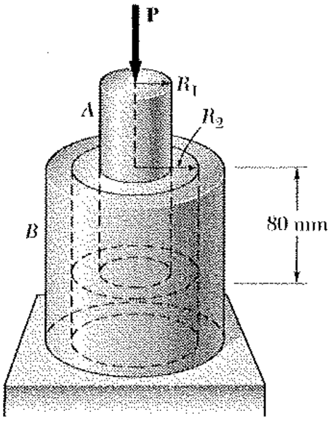
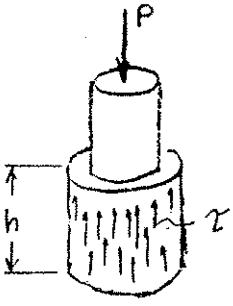
Problem 2.88

***2.88** A vibration isolation support consists of a rod A of radius R_1 and a tube B of inner radius R_2 bonded to a 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 10.93$ MPa. Determine the required value of the ratio R_2/R_1 if a 10-kN force **P** is to cause a 2-mm deflection of rod A.



Let r be a radial coordinate. Over the hollow rubber cylinder $R_1 \le r \le R_2$

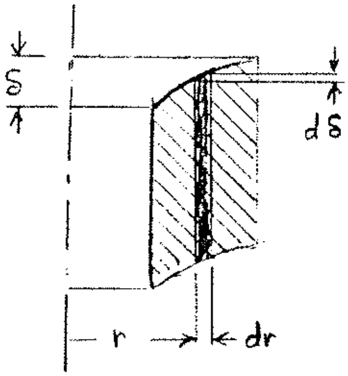


Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$



Shearing deformation over radial length dr

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi G h}\frac{dr}{r}$$

Total deformation

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi G h}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi G h}\ln r\Big|_{R_1}^{R_2} = \frac{P}{2\pi G h}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi G h}\ln\frac{R_2}{R_1}$$

$$\ln\frac{R_2}{R_1} = \frac{2\pi G h \delta}{P} = \frac{(2\pi)(10.93\times10^6)(0.080)(0.002)}{10\times10^3} = 1.0988$$

$$\frac{R_2}{R_1} = \exp(1.0988) = 3.00$$

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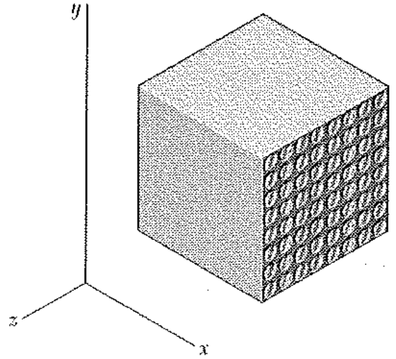
Problem 2.89

***2.89** A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the *x* direction. The cube is constrained against deformations in the *y* and *z* directions and is subjected to a tensile load of 65 kN in the *x* direction. Determine (*a*) the change in the length of the cube in the *x* direction, (*b*) the stresses σ_x, σ_y, and σ_z.

$E_x = 50$ GPa $\nu_{xz} = 0.254$
$E_y = 15.2$ GPa $\nu_{xy} = 0.254$
$E_z = 15.2$ GPa $\nu_{zy} = 0.428$



Stress-to-strain equations are

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \qquad (1) \qquad\qquad \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \qquad (4)$$

$$\epsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \qquad (2) \qquad\qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \qquad (5)$$

$$\epsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \qquad (3) \qquad\qquad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \qquad (6)$$

The constraint conditions are $\varepsilon_y = 0$ and $\varepsilon_z = 0$.

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y}\sigma_y - \frac{\nu_{zy}}{E_z}\sigma_z = \frac{\nu_{xy}}{E_x}\sigma_x \qquad (7)$$

$$-\frac{\nu_{yz}}{E_y}\sigma_y + \frac{1}{E_z}\sigma_z = \frac{\nu_{xz}}{E_x}\sigma_x \qquad (8)$$

$$\frac{1}{15.2}\sigma_y - \frac{0.428}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad \sigma_y - 0.428\sigma_z = 0.077216\,\sigma_x$$

$$-\frac{0.428}{15.2}\sigma_y + \frac{1}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad -0.428\sigma_y + \sigma_z = 0.077216\,\sigma_x$$

Solving simultaneously, $\sigma_y = \sigma_z = 0.134993\,\sigma_x$

Using (4) and (5) in (1), $\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{xy}}{E_x}\sigma_y - \frac{\nu_{xz}}{E}\sigma_z$

$$\varepsilon_x = \frac{1}{E_x}\left[1 - (0.254)(0.134993) - (0.254)(0.134993)\right]\sigma_x$$

$$= \frac{0.93142\,\sigma_x}{E_x}$$

$$A = (40)(40) = 1600\ \text{mm}^2 = 1600 \times 10^{-6}\ \text{m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{65 \times 10^3}{1600 \times 10^{-6}} = 40.625 \times 10^6\ \text{Pa}$$

continued

Problem 2.89 continued

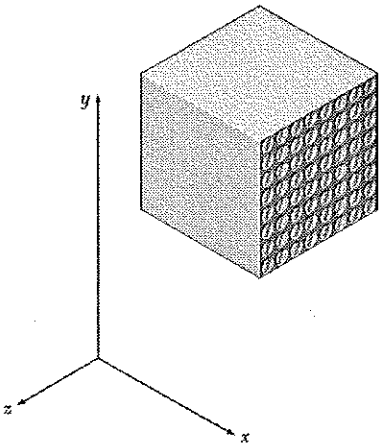
$$\varepsilon_x = \frac{(0.93142)(40.625\times10^3)}{50\times10^9} = 756.78\times10^{-6}$$

(a) $\delta_x = L_x \varepsilon_x = (40\text{ mm})(756.78\times10^{-6}) = 0.0303\text{ mm}$

(b) $\sigma_x = 40.625\times10^6\text{ Pa} = 40.6\text{ MPa}$

$\sigma_y = \sigma_z = (0.134993)(40.625\times10^6) = 5.48\times10^6\text{ Pa}$
$= 5.48\text{ MPa}$

Problem 2.90



***2.90** The composite cube of Prob. 2.89 is constrained against deformation in the *z* direction and elongated in the *x* direction by 0.035 mm due to a tensile load in the *x* direction. Determine (*a*) the stresses σ_x, σ_y, and σ_z, (*b*) the change in the dimension in the *y* direction.

$E_x = 50$ GPa $\quad \nu_{xz} = 0.254$
$E_y = 15.2$ GPa $\quad \nu_{xy} = 0.254$
$E_z = 15.2$ GPa $\quad \nu_{zy} = 0.428$

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \quad (1) \qquad \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \quad (4)$$

$$\epsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \quad (2) \qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \quad (5)$$

$$\epsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \quad (3) \qquad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \quad (6)$$

Constraint condition. $\quad \varepsilon_z = 0$

Load condition. $\quad \sigma_y = 0$

From equation (3), $\quad 0 = -\frac{\nu_{xz}}{E_x}\sigma_x + \frac{1}{E_z}\sigma_z$

$$\sigma_z = \frac{\nu_{xz}E_z}{E_x}\sigma_x = \frac{(0.254)(15.2)}{50} = 0.077216\,\sigma_x$$

continued

Problem 2.90 continued

From equation (1) with $\sigma_y = 0$,

$$\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{zx}}{E_z}\sigma_z = \frac{1}{E_x}\sigma_x - \frac{\nu_{xz}}{E_x}\sigma_z$$

$$= \frac{1}{E_x}\left[\sigma_x - 0.254\,\sigma_z\right] = \frac{1}{E_x}\left[1 - (0.254)(0.077216)\right]\sigma_x$$

$$= \frac{0.98039}{E_x}\sigma_x$$

$$\sigma_x = \frac{E_x\,\varepsilon_x}{0.98039}$$

But, $\varepsilon_x = \dfrac{\delta_x}{L_x} = \dfrac{0.035 \text{ mm}}{40 \text{ mm}} = 875\times10^{-6}$

(a) $\sigma_x = \dfrac{(50\times10^{9})(875\times10^{-6})}{0.98039} = 44.625\times10^{3}\text{ Pa} = 44.6\text{ MPa}$ ◀

$\sigma_y = 0$ ◀

$\sigma_z = (0.077216)(44.625\times10^{6}) = 3.446\times10^{6}\text{ Pa} = 3.45\text{ MPa}$ ◀

From (2), $\varepsilon_y = -\dfrac{\nu_{xy}}{E_x}\sigma_x + \dfrac{1}{E_y}\sigma_y - \dfrac{\nu_{zy}}{E_z}\sigma_z$

$$= -\frac{(0.254)(44.625\times10^{6})}{50\times10^{9}} + 0 - \frac{(0.428)(3.446\times10^{6})}{15.2\times10^{9}}$$

$$= -323.73\times10^{-6}$$

(b) $\delta_y = L_y\,\varepsilon_y = (40\text{ mm})(-323.73\times10^{-6}) = -0.0129\text{ mm}$ ◀

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Problem 2.91

***2.91** The material constants *E, G, k,* and *ν* are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that *(a)* $k = GE/(9G - 3E)$ and *(b)* $\nu = (3k - 2G)/(6k + 2G)$.

$$k = \frac{E}{3(1-2\nu)} \quad \text{and} \quad G = \frac{E}{2(1+\nu)}$$

(a) $1 + \nu = \frac{E}{2G}$ or $\nu = \frac{E}{2G} - 1$

$$k = \frac{E}{3[1 - 2(\frac{E}{2G} - 1]} = \frac{2EG}{3[2G - 2E + 4G} = \frac{2EG}{18G - 6E}$$

$$= \frac{EG}{9G - 6E}$$

(b) $\frac{k}{G} = \frac{2(1+\nu)}{3(1-2\nu)}$

$$3k - 6k\nu = 2G + 2G\nu$$

$$3k - 2G = 2G + 6k$$

$$\nu = \frac{3k - 2G}{6k + 2G}$$

Problem 2.92

***2.92** Show that for any given material, the ratio *G/E* of the modulus of rigidity over the modulus of elasticity is always less than $\frac{1}{2}$ but more than $\frac{1}{3}$. [*Hint*: Refer to Eq. (2.43) and to Sec. 2.13.]

$G = \frac{E}{2(1+\nu)}$ or $\frac{E}{G} = 2(1+\nu)$

Assume $\nu \geq 0$ for almost all materials and $\nu < \frac{1}{2}$ for a positive bulk modulus.

Applying the bounds, $2 \leq \frac{E}{G} < 2(1+\frac{1}{2}) = 3$

Taking the reciprocals, $\frac{1}{2} \geq \frac{G}{E} > \frac{1}{3}$

or $\frac{1}{3} \leq \frac{G}{E} \leq \frac{1}{2}$

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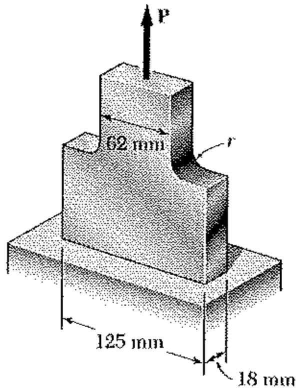
Problem 2.93

2.93 Knowing that $P = 40$ kN, determine the maximum stress when (*a*) $r = 12$ mm, (*b*) $r = 15$ mm.



$P = 40 kN \qquad D = 125 mm \qquad d = 62 mm.$

$\frac{D}{d} = \frac{125}{62} = 2.016$

$A_{min} = dt = (0.062)(0.018) = 1.116 \times 10^{-3}\ m^2$

(a) $r = 12 mm \qquad \frac{r}{d} = \frac{12}{62} = 0.1935$

From Fig. 2.64 b $\quad K = 1.94$

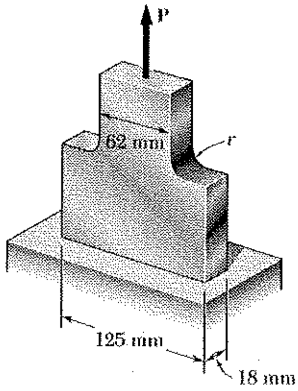
$\sigma_{max} = \frac{KP}{A_{min}} = \frac{(1.94)(40 \times 10^3)}{1.116 \times 10^{-3}} = 69.5\ MPa.$

(b) $r = 15 mm \qquad \frac{r}{d} = \frac{15}{62} = 0.24 \qquad K = 1.86.$

$\sigma_{max} = \frac{KP}{A_{min}} = \frac{(1.86)(40 \times 10^3)}{1.116 \times 10^{-3}} = 66.7\ MPa.$

Problem 2.94

2.94 Knowing that, for the plate shown, the allowable stress is 110 MPa, determine the maximum allowable value of P when (*a*) $r = 10$ mm, (*b*) $r = 18$ mm.



$\sigma_{max} = 110\ MPa$

$D = 125 mm \qquad d = 62 mm \qquad \frac{D}{d} = 2.016.$

$A_{min} = dt = (0.062)(0.018) = 1.116 \times 10^{-3}\ m^2.$

(a) $r = 10 mm \qquad \frac{r}{d} = \frac{10}{62} = 0.161$

From Fig. 2.64 b $\quad K = 2.09$

$\sigma_{max} = \frac{KP}{A_{min}}$

$P = \frac{A_{min}\sigma_{max}}{K} = \frac{(1.116 \times 10^{-3})(110 \times 10^6)}{2.09} = 58.7\ kN.$

(b) $r = 18 mm \qquad \frac{r}{d} = \frac{18}{062} = 0.29 \qquad K = 1.76$

$P = \frac{A_{min}\sigma_{max}}{K} = \frac{(1.116 \times 10^{-3})(110 \times 10^6)}{1.76} = 69.75\ kN.$

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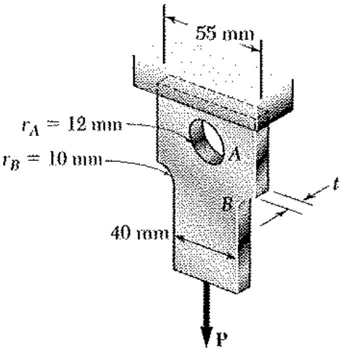
Problem 2.95

2.95 For $P = 35$ kN, determine the minimum plate thickness t required if the allowable stress is 125 MPa.



At the hole: $r_A = 12\text{mm}$ $d_A = 55 - 24 = 31$

$$\frac{r_A}{d_A} = \frac{12}{31} = 0.39$$

From Fig 2.64 a $K = 2.26$

$$\sigma_{max} = \frac{KP}{A_{net}} = \frac{KP}{d_A t} \therefore t = \frac{KP}{d_A \sigma_{max}}$$

$$t = \frac{(2.26)(35000)}{(0.031)(125\times10^6} = 0.0204\text{ m} = 20.4\text{mm}$$

At the fillet $D = 55\text{mm}$ $d_B = 40\text{mm}$ $\frac{D}{d_B} = \frac{55}{40} = 1.375$

$r_B = \frac{3}{8} = 10\text{mm}$ $\frac{r_B}{d_B} = \frac{10}{40} = 0.25$

From Fig 2.64 b $K = 1.70$ $\sigma_{max} = \frac{KP}{A_{min}} = \frac{KP}{d_B t}$

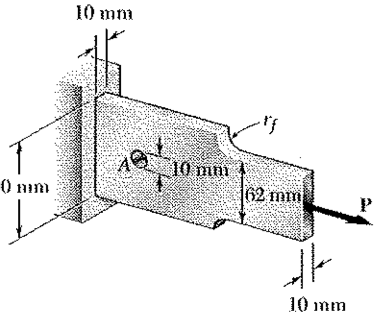
$$t = \frac{KP}{d_B \sigma_{max}} = \frac{(1.70)(35000)}{(0.04)(125\times10^6)} = 0.0119\text{ m} = 11.9\text{mm}$$

The larger value is the required minimum plate thickness

$t = 20.4\text{mm}$

Problem 2.96

2.96 Knowing that the hole has a diameter of 10 mm, determine (*a*) the radius r_f of the fillets for which the same maximum stress occurs at the hole *A* and at the fillets, (*b*) the corresponding maximum allowable load **P** if the allowable stress is 105 MPa.



For the circular hole $r = \frac{10}{2} = 5\text{mm}$

$d = 100 - 10 = 90\text{mm}$ $\frac{r}{d} = \frac{5}{90} = 0.056$

$A_{net} = d\,t = (0.09)(0.01) = 0.0009\text{ m}^2$

From Fig 2.64 a $K_{hole} = 2.8$

$$\sigma_{max} = \frac{K_{hole} P}{A_{net}}$$

(b) $P = \frac{A_{net}\sigma_{max}}{K_{hole}} = \frac{(0.0009)(105\times10^6)}{2.8} = 33.75\text{kN}$

(a) For fillet $D = 100\text{mm}$ $d = 62\text{mm}$ $\frac{D}{d} = \frac{100}{62} = 1.61$

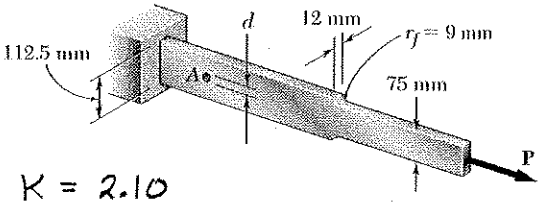
$A_{min} = dt = (0.062)(0.01) = 0.00062\text{ m}^2$

$$\sigma_{max} = \frac{K_{fillet} P}{A_{min}} \therefore K_{fillet} = \frac{A_{min}\sigma_{max}}{P} \approx \frac{(0.00062)(105\times10^6)}{33750} = 1.93$$

From Fig 2.64 b $\frac{r_f}{d} \approx 0.18 \therefore r_f \approx 0.18d = (0.18)(62) \approx 11.16\text{ mm}$

Problem 2.97

2.97 A hole is to be drilled in the plate at *A*. The diameters of the bits available to drill the hole range from 12 to 24 mm in 3-mm increments. (*a*) Determine the diameter *d* of the largest bit that can be used if the allowable load at the hole is to exceed that at the fillets. (*b*) If the allowable stress in the plate is 145 MPa, what is the corresponding allowable load **P**?



At the fillets, $r = 9$ mm $d = 75$ mm

$D = 112.5$ mm $\frac{D}{d} = \frac{112.5}{75} = 1.5$

$\frac{r}{d} = \frac{9}{75} = 0.12$ From Fig 2.64 b $K = 2.10$

$A_{min} = (75)(12) = 900\ \text{mm}^2 = 900\ \text{mm}^2 = 900 \times 10^{-6}\ \text{m}^2$

$$\sigma_{max} = K \frac{P_{all}}{A_{min}} = \sigma_{all} \therefore P_{all} = \frac{A_{min}\sigma_{all}}{K} = \frac{(900 \times 10^{-6})(145 \times 10^{6})}{2.10}$$

$$= 62.1 \times 10^{3}\ \text{N} = 62.1\ \text{kN}$$

At the hole, $A_{net} = (D - 2r)\,t$, $\frac{r}{d} = \frac{r}{D-2r}$

where $D = 112.5$ mm $r =$ radius of circle $t = 12$ mm

K is taken from Fig 2.64 a.

$$\sigma_{max} = K \frac{P}{A_{net}} = \sigma_{all} \therefore P_{all} = \frac{A_{net}\sigma_{all}}{K}$$

Hole diam	r	$d = D - 2r$	r/d	K	$A_{net}\ (\text{m}^2)$	P_{all}, (N)
12 mm	6.0 mm	100.5	0.060	2.80	1206×10^{-6}	62.5×10^{3}
15 mm	7.5 mm	97.5 mm	0.077	2.75	1170×10^{-6}	61.7×10^{3}
18 mm	9.0 mm	94.5 mm	0.095	2.71	1134×10^{-6}	60.7×10^{3}
21 mm	10.5 mm	91.5 mm	0.115	2.67	1098×10^{-6}	59.6×10^{3}
24 mm	12.0 mm	88.5 mm	0.136	2.62	1062×10^{-6}	58.8×10^{3}

(a) Bit diameter, 1 = 12 mm ◀

(b) Allowable load. 62.1 kN ◀

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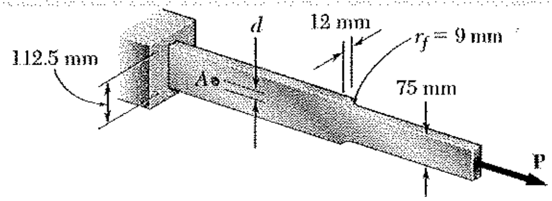
Problem 2.98

2.98 (*a*) For $P = 58$ kN and $d = 12$ mm, determine the maximum stress in the plate shown. (*b*) Solve part *a*, assuming that the hole at *A* is not drilled.



Maximum stress at hole.

Use Fig. 2.64 a for values of K

$$\frac{r}{d} = \frac{6}{112.5-12} = 0.0597, \qquad K = 2.80$$

$$A_{net} = (12)(112.5-12) = 1206 \text{ mm}^2 = 1206 \times 10^{-6} \text{ m}^2$$

$$\sigma_{max} = K\frac{P}{A_{net}} = \frac{(2.80)(58\times10^3)}{1206\times10^{-6}} = 134.7\times10^6 \text{ Pa}$$

Maximum stress at fillets

Use Fig. 2.64 b

$$\frac{r}{d} = \frac{9}{75} = 0.12, \qquad \frac{D}{d} = \frac{112.5}{75} = 1.50, \qquad K = 2.10$$

$$A_{min} = (12)(75) = 900 \text{ mm}^2 = 900\times10^{-6} \text{ m}^2$$

$$\sigma_{max} = K\frac{P}{A_{min}} = \frac{(2.10)(58\times10^3)}{900\times10^{-6}} = 135.3\times10^6 \text{ Pa}$$

(a) With hole and fillets. $\sigma_{max} = 134.7$ MPa

(b) Without hole. $\sigma_{max} = 135.3$ MPa

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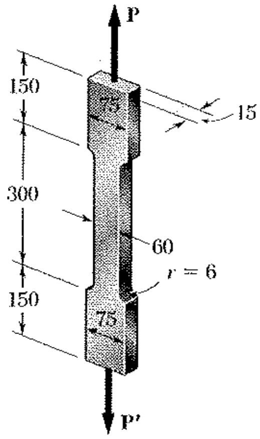
Problem 2.99

2.99 The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude P. (*a*) Knowing that $E = 70$ GPa and $\sigma_{all} = 200$ MPa, determine the maximum allowable value of P and the corresponding total elongation of the specimen. (*b*) Solve part *a*, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60 × 15-mm rectangular cross section.



Dimensions in mm

$\sigma_{all} = 200 \times 10^6$ Pa $\qquad E = 70 \times 10^9$ Pa

$A_{min} = (60\text{ mm})(15\text{ mm}) = 900\text{ mm}^2 = 900 \times 10^{-6}\text{ m}^2$

(a) Test specimen. $\qquad D = 75$ mm, $d = 60$ mm, $r = 6$ mm

$$\frac{D}{d} = \frac{75}{60} = 1.25 \qquad \frac{r}{d} = \frac{6}{60} = 0.10$$

From Fig. 2.64(b) $\qquad K = 1.95 \qquad \sigma_{max} = K\frac{P}{A}$

$$P = \frac{A\sigma_{max}}{K} = \frac{(900\times10^{-6})(200\times10^6)}{1.95} = 92.308\times10^3\text{ N}$$

$P = 92.3$ kN

Wide area. $\quad A^* = (75\text{ mm})(15\text{ mm}) = 1125\text{ mm}^2 = 1.125\times10^{-3}\text{ m}^2$

$$\delta = \sum \frac{P_i L_i}{A_i E_i} = \frac{P}{E}\sum\frac{L_i}{A_i} = \frac{92.308\times10^3}{70\times10^9}\left[\frac{0.150}{1.125\times10^{-3}} + \frac{0.300}{900\times10^{-6}} + \frac{0.150}{1.125\times10^{-3}}\right]$$

$$= 791\times10^{-6}\text{ m}$$

$\delta = 0.791$ mm

(b) Uniform bar.

$P = A\sigma_{all} = (900\times10^{-6})(200\times10^6) = 180\times10^3$ N $\qquad P = 180.0$ kN

$$\delta = \frac{PL}{AE} = \frac{(180\times10^3)(0.600)}{(900\times10^{-6})(70\times10^9)} = 1.714\times10^{-3}\text{ m}$$

$\delta = 1.714$ mm

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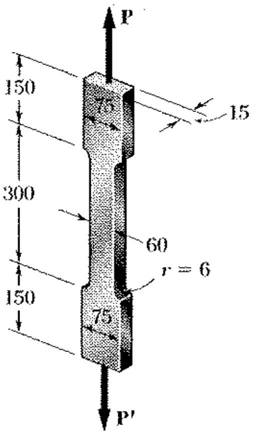
Problem 2.100

2.100 For the test specimen of Prob. 2.99, determine the maximum value of the normal stress corresponding to a total elongation of 0.75 mm.



Dimensions in mm

2.99 The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude P. (*a*) Knowing that $E = 70$ GPa and $\sigma_{all} = 200$ MPa, determine the maximum allowable value of P and the corresponding total elongation of the specimen. (*b*) Solve part *a*, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60 × 15-mm rectangular cross section.

$$\delta = \Sigma \frac{P_i L_i}{E_i A_i} = \frac{P}{E} \Sigma \frac{L_i}{A_i} \qquad \delta = 0.75 \times 10^{-3}\ \text{m}$$

$$L_1 = L_3 = 150\ \text{mm} = 0.150\ \text{m}, \quad L_2 = 300\ \text{mm} = 0.300\ \text{m}$$

$$A_1 = A_3 = (75\ \text{mm})(15\ \text{mm}) = 1125\ \text{mm}^2 = 1.125 \times 10^{-3}\ \text{m}^2$$

$$A_2 = (60\ \text{mm})(15\ \text{mm}) = 900\ \text{mm}^2 = 900 \times 10^{-6}\ \text{m}^2$$

$$\Sigma \frac{L_i}{A_i} = \frac{0.150}{1.125 \times 10^{-3}} + \frac{0.300}{900 \times 10^{-6}} + \frac{0.150}{1.125 \times 10^{-3}} = 600\ \text{m}^{-1}$$

$$P = \frac{E\delta}{\Sigma \frac{L_i}{A_i}} = \frac{(70 \times 10^9)(0.75 \times 10^{-3})}{600} = 87.5 \times 10^3\ \text{N}$$

Stress concentration. $D = 75\ \text{mm}, \quad d = 60\ \text{mm}, \quad r = 6\ \text{mm}$

$$\frac{D}{d} = \frac{75}{60} = 1.25 \qquad \frac{r}{d} = \frac{6}{60} = 0.10$$

From fig. 2.59 $\quad K = 1.95$

$$\sigma_{max} = K \frac{P}{A_{min}} = \frac{(1.95)(87.5 \times 10^3)}{900 \times 10^{-6}} = 189.6 \times 10^6\ \text{Pa} \qquad \sigma_{max} = 189.6\ \text{MPa} \blacktriangleleft$$

Note that $\sigma_{max} \leq \sigma_{all}$.

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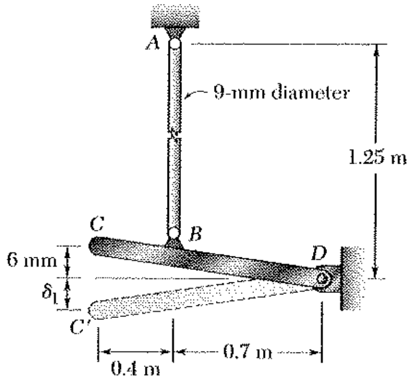
Problem 2.101



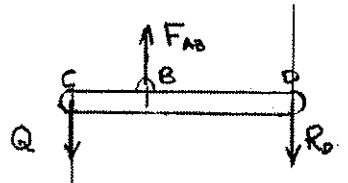
2.101 Rod *AB* is made of a mild steel that is assumed to be elastoplastic with E = 200 GPa and σ_Y = 345 MPa. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is 6 mm too high. A vertical force **Q** is then applied at *C* until this point has moved to position *C'*. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

$A_{AB} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617\times10^{-6} \text{ m}^2$

Since rod AB is to be stretched permanently,

$(F_{AB})_{max} = A_{AB}\sigma_Y = (63.617\times10^{-6})(345\times10^{6})$

$= 21.948\times10^{3} \text{ N}$

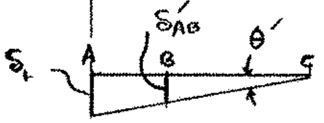


$+\circlearrowleft \Sigma M_D = 0: \quad 1.1\,Q - 0.7\,F_{AB} = 0$

$Q_{max} = \frac{0.7}{1.1}(21.948\times10^{3}) = 13.967\times10^{3} \text{ N}$

13.97 kN ◄

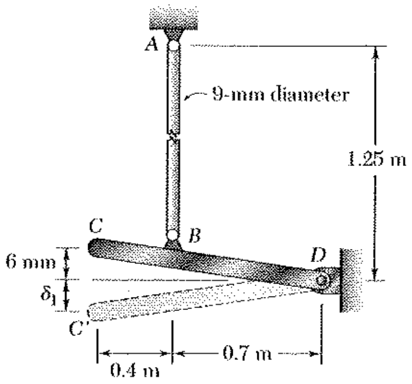
$\delta'_{AB} = \frac{(F_{AB})_{max}L_{AB}}{EA_{AB}} = \frac{(21.948\times10^{3})(1.25)}{(200\times10^{9})(63.617\times10^{-6})} = 2.15625\times10^{-3} \text{ m}$



$\theta' = \frac{\delta'_{AB}}{0.7} = 3.0804\times10^{-3} \text{ rad}$

$\delta_1 = 1.1\,\theta' = 3.39\times10^{-3} \text{ m}$ 3.39 mm ◄

Problem 2.102



2.102 Solve Prob. 2.101, assuming that the yield point of the mild steel is 250 MPa.

2.101 Rod *AB* is made of a mild steel that is assumed to be elastoplastic with E = 200 GPa and σ_Y = 345 MPa. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is 6 mm too high. A vertical force **Q** is then applied at *C* until this point has moved to position *C'*. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

$A_{AB} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617\times10^{-6} \text{ m}^2$

Since rod AB is to be stretched permanently,

$(F_{AB})_{max} = A_{AB}\sigma_Y = (63.617\times10^{-6})(250\times10^{6})$

$= 15.9043\times10^{3} \text{ N}$

$+\circlearrowleft \Sigma M_D = 0 \quad 1.1\,Q - 0.7\,F_{AB} = 0$

$Q_{max} = \frac{0.7}{1.1}(15.9043\times10^{3}) = 10.12\times10^{3} \text{ N}$

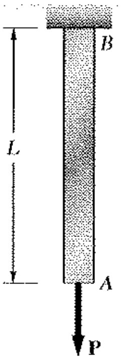
10.12 kN ◄

$\delta'_{AB} = \frac{(F_{AB})_{max}L_{AB}}{EA_{AB}} = \frac{(15.9043\times10^{3})(1.25)}{(200\times10^{9})(63.617\times10^{-6})} = 1.5625\times10^{-3} \text{ m}$

$\theta' = \frac{\delta'_{AB}}{0.7} = 2.2321\times10^{-3} \text{ rad}$

$\delta_1 = 1.1\,\theta' = 2.46\times10^{-3} \text{ m}$ 2.46 mm ◄

Problem 2.103



2.103 The 30-mm square bar *AB* has a length $L = 2.2$ m; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force **P** is applied to the bar until end *A* has moved down by an amount δ_m. Determine the maximum value of the force **P** and the permanent set of the bar after the force has been removed, knowing that (*a*) $\delta_m = 4.5$ mm, (*b*) $\delta_m = 8$ mm.

$$A = (30)(30) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(2.2)(345 \times 10^6)}{200 \times 10^9} = 3.795 \times 10^{-3} = 3.795 \text{ mm}$$

$$\text{If } \delta_m \geq \delta_Y \qquad P_m = A\sigma_Y = (900 \times 10^{-6})(345 \times 10^6) = 310.5 \times 10^3 \text{ N}$$

$$\underline{\text{Unloading}}: \ \delta' = \frac{P_m L}{AE} = \frac{\sigma_Y L}{E} = \delta_Y = 3.795 \text{ mm}$$

$$\delta_f = \delta_m - \delta'$$

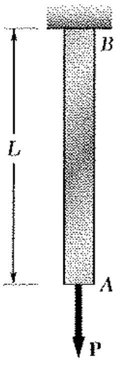
(a) $\delta_m = 4.5 \text{ mm} > \delta_Y \qquad P_m = 310.5 \times 10^3 \text{ N} = 310.5 \text{ kN}$ ◀

$\delta_{perm} = 4.5 \text{ mm} - 3.795 \text{ mm} = 0.705 \text{ mm}$ ◀

(b) $\delta_m = 8 \text{ mm} > \delta_Y \qquad P_m = 310.5 \times 10^3 \text{ N} = 310.5 \text{ kN}$ ◀

$\delta_p = 8.0 \text{ mm} - 3.795 \text{ mm} = 4.205 \text{ mm}$ ◀

Problem 2.104



2.104 The 30-mm square bar *AB* has a length $L = 2.5$ m; it is made of mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force **P** is applied to the bar and then removed to give it a permanent set δ_p. Determine the maximum value of the force **P** and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_p is (*a*) 3.5 mm, (*b*) 6.5 mm.

$$A = (30)(30) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(2.5)(345 \times 10^6)}{200 \times 10^9} = 4.3125 \times 10^{-3} \text{ m} = 4.3125 \text{ mm}$$

When δ_m exceeds δ_Y, thus producing a permanent stretch of δ_p, the maximum force is

$$P_m = A\sigma_Y = (900 \times 10^{-6})(345 \times 10^6) = 310.5 \times 10^3 \text{ N} = 310.5 \text{ kN}$$ ◀

$$\delta_p = \delta_m - \delta' = \delta_m - \delta_Y \quad \therefore \quad \delta_m = \delta_p + \delta_Y$$

(a) $\delta_p = 3.5 \text{ mm} \qquad \delta_m = 3.5 \text{ mm} + 4.3125 \text{ mm} = 7.81 \text{ mm}$ ◀

(b) $\delta_p = 6.5 \text{ mm} \qquad \delta_m = 6.5 \text{ mm} + 4.3125 \text{ mm} = 10.81 \text{ mm}$ ◀

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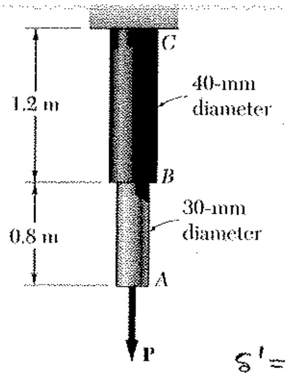
Problem 2.105



2.105 Rod *ABC* consists of two cylindrical portions *AB* and *BC*; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod and then removed to give it a permanent set $\delta_p = 2$ mm. Determine the maximum value of the force **P** and the maximum amount δ_m by which the rod should be stretched to give it the desired permanent set.

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\ \text{m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664\times10^{3}\ \text{mm}^2 = 1.25664\times10^{-3}\ \text{m}^2$$

$$P_{max} = A_{min}\sigma_Y = (706.86\times10^{-6})(250\times10^{6}) = 176.715\times10^{3}\ \text{N}$$

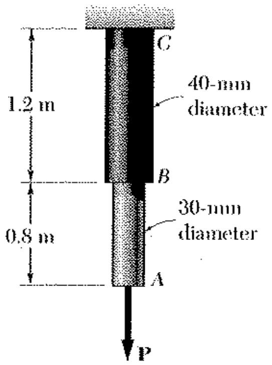
$$P_{max} = 176.7\ \text{kN} \blacktriangleleft$$

$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715\times10^{3})(0.8)}{(200\times10^{9})(706.86\times10^{-6})} + \frac{(176.715\times10^{3})(1.2)}{(200\times10^{9})(1.25664\times10^{-3}}$$

$$= 1.84375\times10^{-3}\ \text{m} = 1.84375\ \text{mm}$$

$$\delta_p = \delta_m - \delta' \quad \text{or} \quad \delta_m = \delta_p + \delta' = 2 + 1.84375 \qquad \delta_m = 3.84\ \text{mm} \blacktriangleleft$$

Problem 2.106



2.106 Rod *ABC* consists of two cylindrical portions *AB* and *BC*; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod until its end *A* has moved down by an amount $\delta_m = 5$ mm. Determine the maximum value of the force **P** and the permanent set of the rod after the force has been removed.

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\ \text{m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664\times10^{3}\ \text{mm}^2 = 1.25644\times10^{-3}\ \text{m}^2$$

$$P_{max} = A_{min}\sigma_Y = (706.86\times10^{-6})(250\times10^{6}) = 176.715\times10^{3}\ \text{N}$$

$$P_{max} = 176.7\ \text{kN} \blacktriangleleft$$

$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715\times10^{3})(0.8)}{(200\times10^{9})(706.68\times10^{-6})} + \frac{(176.715\times10^{3})(1.2)}{(200\times10^{9})(1.25664\times10^{-3})}$$

$$= 1.84375\times10^{-3}\ \text{m} = 1.84375\ \text{mm}$$

$$\delta_p = \delta_m - \delta' = 5 - 1.84375 = 3.16\ \text{mm} \qquad \delta_p = 3.16\ \text{mm} \blacktriangleleft$$

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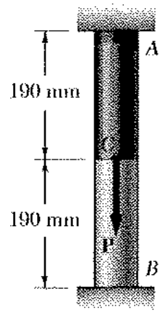
Problem 2.107



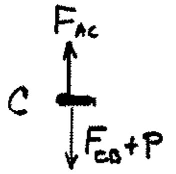
2.107 Rod *AB* consists of two cylindrical portions *AC* and *BC*, each with a cross-sectional area of 1750 mm². Portion *AC* is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion *CB* is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load **P** is applied at *C* as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of *C* if *P* is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of *C*.

Displacement at C to cause yielding of AC

$$\delta_{C,Y} = L_{AC}\,\varepsilon_{Y,AC} = \frac{L_{AC}\,\sigma_{Y,AC}}{E} = \frac{(0.190)(250\times10^6)}{200\times10^9} = 0.2375\times10^{-3}\ \text{m}$$

Corresponding force $\quad F_{AC} = A\,\sigma_{Y,AC} = (1750\times10^{-6})(250\times10^6)$
$= 437.5\times10^3\ \text{N}$

$$F_{CB} = -\frac{EA\delta_C}{L_{CB}} = -\frac{(200\times10^9)(1750\times10^{-6})(0.2375\times10^{-3})}{0.190} = -437.5\times10^3\ \text{N}$$



For equilibrium of element at C

$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 875\times10^3\ \text{N}$$

Since applied load $P = 975\times10^3\ \text{N} > 875\times10^3\ \text{N}$, portion AC yields.

$$F_{CB} = F_{AC} - P = 437.5\times10^3 - 975\times10^3\ \text{N} -= -537.5\times10^3\ \text{N}$$

(a) $\delta_C = -\dfrac{F_{CB}L_{CB}}{EA} = \dfrac{(537.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.29179\times10^{-3}\ \text{m}$
$= 0.292\ \text{mm}$

(b) Maximum stresses: $\quad \sigma_{AB} = \sigma_{Y,AB} = 250\ \text{MPa}$

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{537.5\times10^3}{1750\times10^{-6}} = -307.14\times10^6\ \text{Pa} = -307\ \text{MPa}$$

(c) Deflection and forces for unloading.

$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore\ P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{CB}} = -P'_{AC}$$

$$P' = 975\times10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \qquad P'_{AC} = 487.5\times10^{3}\ \text{N}$$

$$\delta' = \frac{(487.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.26464\times10^{3}\ \text{m}$$

$$\delta_p = \delta_m - \delta' = 0.29179\times10^{-3} - 0.26464\times10^{-3} = 0.02715\times10^{-3}\ \text{m}$$

$= 0.027\ \text{mm}$

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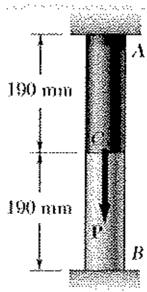
Problem 2.108



2.108 For the composite rod of Prob. 2.107, if *P* is gradually increased from zero until the deflection of point *C* reaches a maximum value of $\delta_m = 0.03$ mm and then decreased back to zero, determine, (*a*) the maximum value of *P*, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of *C* after the load is removed.

2.107 Rod *AB* consists of two cylindrical portions *AC* and *BC*, each with a cross-sectional area of 1750 mm^2. Portion *AC* is made of a mild steel with E = 200 GPa and σ_Y = 250 MPa, and portion *CB* is made of a high-strength steel with E = 200 GPa and σ_Y = 345 MPa. A load **P** is applied at *C* as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of *C* if *P* is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of *C*.

Displacement at C is $\delta_m = 0.30$ mm. The corresponding strains are

$$\varepsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.30 \text{ mm}}{190 \text{ mm}} = 1.5789 \times 10^{-3}$$

$$\varepsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.30 \text{ mm}}{190 \text{ mm}} = -1.5789 \times 10^{-3}$$

Strains at initial yielding

$$\varepsilon_{Y,AC} = \frac{\sigma_{Y,AC}}{E} = \frac{250 \times 10^6}{200 \times 10^9} = 1.25 \times 10^{-3} \qquad \text{(yielding)}$$

$$\varepsilon_{Y,CB} = -\frac{\sigma_{Y,BC}}{E} = -\frac{345 \times 10^6}{200 \times 10^9} = -1.725 \times 10^{-3} \qquad \text{(elastic)}$$

(a) Forces: $F_{AC} = A\sigma_Y = (1750 \times 10^{-6})(250 \times 10^6) = 437.5 \times 10^{-3}$ N

$F_{CB} = EA\varepsilon_{CB} = (200 \times 10^9)(1750 \times 10^{-6})(-1.5789 \times 10^{-3}) = -552.6 \times 10^{-3}$ N

For equilibrium of element at C, $F_{AC} - F_{CB} - P = 0$

$P = F_{AC} - F_{CD} = 437.5 \times 10^3 + 552.6 \times 10^3 = 990.1 \times 10^3 \text{ N} = 990 \text{ kN}$ ◄

(b) Stresses: AC $\sigma_{AC} = \sigma_{Y,AC} = 250$ MPa ◄

CB $\epsilon_{CB} = \dfrac{F_{CB}}{A} = -\dfrac{552.6 \times 10^3}{1750 \times 10^{-6}} = -316 \times 10^6 \text{ Pa} = -316 \text{ MPa}$ ◄

(c) Deflection and forces for unloading.

$$\delta' = \frac{P'_{AC} L_{AC}}{EA} = -\frac{P'_{CB} L_{CB}}{EA} \quad \therefore \quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P_{AC}$$

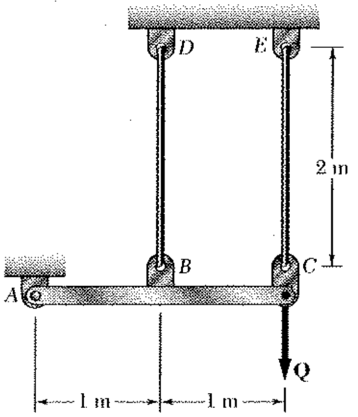
$$P' = P'_{AC} - P'_{CB} = 2P'_{AC} = 990.1 \times 10^3 \text{ N} \quad \therefore \quad P'_{AC} = 495.05 \times 10^3 \text{ N}$$

$$\delta' = \frac{(495.05 \times 10^3)(0.190)}{(200 \times 10^9)(1750 \times 10^{-6})} = 0.26874 \times 10^{-3} \text{ m} = 0.26874 \text{ mm}$$

$\delta_p = \delta_m - \delta' = 0.30 \text{ mm} - 0.26874 \text{ mm} = 0.031 \text{ mm}$ ◄

Problem 2.109

2.109 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at C to the rigid bar ABC and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable BD, (*b*) the maximum deflection of point C, (*c*) the final displacement of point C. (*Hint*: In part *c*, cable CE is not taut.)

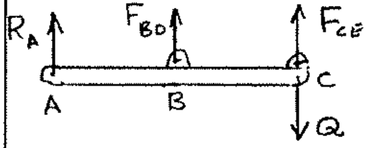


Elongation constraints for taut cables.

Let θ = rotation angle of rigid bar ABC

$$\theta = \frac{\delta_{BD}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\,\delta_{CE} = \frac{1}{2}\,\delta_{CE} \qquad (1)$$



Equilibrium of bar ABC.

$$+\circlearrowleft M_A = 0: \quad L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \qquad (2)$$

Assume cable CE is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}$ N

From (2), $F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^{3} - 34.5\times10^{3}) = 31.0\times10^{3}$ N

Since $F_{BD} \leq A\sigma_Y = 34.5\times10^{3}$ N, cable BD is elastic when $Q = 50$ kN.

(a) Maximum stresses. $\qquad \sigma_{CE} = \sigma_Y = 345$ MPa

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times10^{3}}{100\times10^{-6}} = 310\times10^{6}\ \text{Pa} \qquad \sigma_{BD} = 310\ \text{MPa}$$ ◄

(b) Maximum of deflection of point C.

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = \frac{(31.0\times10^{3})(2)}{(200\times10^{9})(100\times10^{-6})} = 3.1\times10^{-3}\ \text{m}$$

From (1) $\quad \delta_C = \delta_{CE} = 2\,\delta_{BD} = 6.2\times10^{-3}$ m $\qquad$ 6.20 mm ↓ ◄

Permanent elongation of cable CE: $\quad (\delta_{CE})_p = (\delta_{CE}) - \dfrac{\sigma_Y L_{CE}}{E}$

$$(\delta_{CE})_p = (\delta_{CE})_{max} - \frac{F_{CE}L_{CE}}{EA} = (\delta_{CE})_{max} - \frac{\sigma_Y L_{CE}}{E} =$$

$$= 6.20\times10^{-3} - \frac{(345\times10^{6})(2)}{200\times10^{9}} = 2.75\times10^{-3}\ \text{m}$$

(c) Unloading. Cable CE is slack ($F_{CE} = 0$) at $Q = 0$.

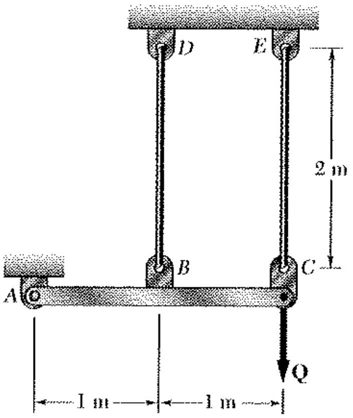
From (2), $\quad F_{BD} = 2(Q - F_{CE}) = 2(0-0) = 0$

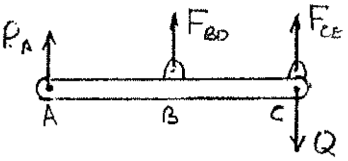
Since cable BD remained elastic, $\quad \delta_{BD} = \dfrac{F_{BD}L_{BD}}{EA} = 0$ ◄

Problem 2.110

2.110 Solve Prob. 2.109, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.

2.109 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at *C* to the rigid bar *ABC* and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable *BD*, (*b*) the maximum deflection of point *C*, (*c*) the final displacement of point *C*. (*Hint*: In part *c*, cable *CE* is not taut.)





Elongation constraints.

Let θ = rotation angle of rigid bar ABC.

$$\theta = \frac{\delta_{BC}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\delta_{CE} = \frac{1}{2}\delta_{CE} \qquad (1)$$

Equilibrium of bar ABC.

$$+\circlearrowleft \Sigma M_A = 0: \quad L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \qquad (2)$$

Assume cable CE is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}$ N

From (2) $F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^{3} - 34.5\times10^{3}) = 31.0\times10^{3}$ N

Since $F_{BD} < A\sigma_Y = 34.5\times10^{3}$ N, cable BD is elastic when $Q = 50$ kN.

(a) Maximum stresses. $\sigma_{CE} = \sigma_Y = 345$ MPa

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times10^{3}}{100\times10^{-6}} = 310\times10^{6} \text{ Pa}$$

$\sigma_{BD} = 310$ MPa ◄

(b) Maximum deflection of point C.

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = \frac{(31.0\times10^{3})(2)}{(200\times10^{9})(100\times10^{-6})} = 3.1\times10^{-3} \text{ m}$$

From (1) $(\delta_C)_m = \delta_{CE} = 2\delta_{BD} = 6.2\times10^{-3}$ m 6.20 mm↓ ◄

Unloading. $Q' = 50\times10^{3}$ N, $\delta_{CE}' = \delta_C'$ From (1) $\delta_{BD}' = \frac{1}{2}\delta_C'$

Elastic $$F_{BD}' = \frac{EA\delta_{BD}'}{L_{BD}} = \frac{(200\times10^{9})(100\times10^{-6})(\frac{1}{2}\delta_C')}{2} = 5\times10^{6}\,\delta_C'$$

$$F_{CE}' = \frac{EA\delta_{CE}'}{L_{CE}} = \frac{(200\times10^{9})(100\times10^{-6})(\delta_C')}{2} = 10\times10^{6}\,\delta_C'$$

From (2) $Q' = F_{CE}' + \frac{1}{2}F_{BD}' = 12.5\times10^{6}\,\delta_C'$

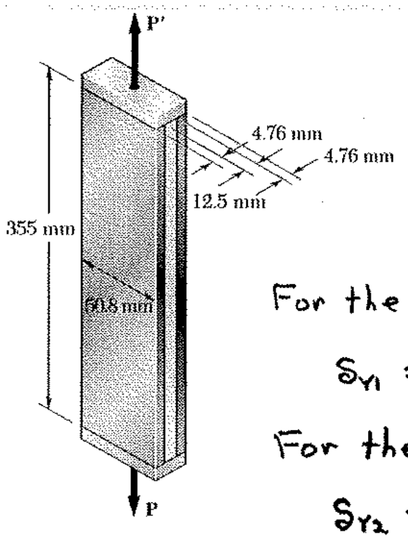
Equating expressions for Q' $12.5\times10^{6}\,\delta_C' = 50\times10^{3}$

$\delta_C' = 4\times10^{-3}$ m

(c) Final displacement. $\delta_C = (\delta_C)_m - \delta_C' = 6.2\times10^{-3} - 4\times10^{-3} = 2.2\times10^{-3}$ m

2.20 mm↓ ◄

Problem 2.111



2.111 Two tempered-steel bars, each 4.76 mm thick, are bonded to a 12.5-mm mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 200$ GPa and with yield strengths equal to 690 MPa and 345 MPa, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 1.016$ mm and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

For the mild steel $\quad A_1 = (0.0125)(0.0508) = 635\times10^{-6}\ m^2$

$$\delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(0.355)(345\times10^6)}{200\times10^9} = 0.612\times10^{-3}\ m$$

For the tempered steel $\quad A_2 = 2(0.00476)(0.0508) = 483.6\times10^{-6}\ m^2$

$$\delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(0.355)(690\times10^6)}{200\times10^9} = 1.225\times10^{-3}\ m.$$

Total area: $\quad A = A_1 + A_2 = 1118.6\times10^{-6}\ m^2.$

$\delta_{Y1} < \delta_m < \delta_{Y2}$ The mild steel yields. Tempered steel is elastic.

(a) Forces $\quad P_1 = A_1\sigma_{Y1} = (635\times10^{-6})(345\times10^6) = 219.075$ kN.

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(200\times10^9)(483.6\times10^{-6})(0.001016)}{0.355} = 276.81\ kN.$$

$P = P_1 + P_2 = 495.9$ kN ◄

(b) Stresses $\quad \sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 345$ MPa

$$\sigma_2 = \frac{P_2}{A_2} = \frac{276810}{483.6\times10^{-6}} = 572.4\ MPa$$ ◄

Unloading $\quad \delta' = \frac{PL}{EA} = \frac{(495900)(0.355)}{(200\times10^9)(1118.6\times10^{-6})} = 0.787\times10^{-3}$ m.

(c) Permanent set $\quad \delta_p = \delta_m - \delta' = 1.016 - 0.787 = 0.229$ mm

$= 0.23$ mm ◄

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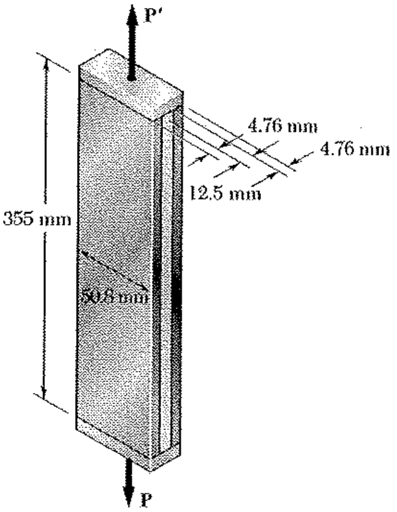
Problem 2.112

2.112 For the composite bar of Prob. 2.111, if *P* is gradually increased from zero to 436 kN and then decreased back to zero, determine (*a*) the maximum deformation of the bar, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

2.111 Two tempered-steel bars, each 4.76 mm thick, are bonded to a 12.5-mm mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude *P*. Both steels are elastoplastic with $E = 200$ GPa and with yield strengths equal to 690 MPa and 345 MPa, respectively, for the tempered and mild steel. The load *P* is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 1.016$ mm and then decreased back to zero. Determine (*a*) the maximum value of *P*, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.



Mild steel $A_1 = (0.0125)(0.0508) = 635\times10^{-6}\ m^2$

Tempered steel $A_2 = 2(0.00476)(0.0508) = 483.6\times10^{-6}\ m^2$

Total: $A = A_1 + A_2 = 1118.6\times10^{-6}\ m^2$

orce to yield the mild steel

$$\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1118.6\times10^{-6})(345\times10^{6}) = 385.9\ kPa$$

$P > P_Y$, therefore mild steel yields.

Let P_1 = force carried by mild steel
P_2 = force carried by tempered steel

$$P_1 = A_1\sigma_1 = (635\times10^{-6})(345\times10^{6}) = 219075\ N$$

$$P_1 + P_2 = P \qquad P_2 = P - P_1 = 436000 - 219075 = 216925\ N$$

(a) $$\delta_m = \frac{P_2 L}{EA_2} = \frac{(216925)(0.355)}{(200\times10^{9})(483.6\times10^{-6})} = 0.796\times10^{-3}\ m = 0.8\ mm$$ ◄

(b) $$\sigma_2 = \frac{P_2}{A_2} = \frac{(216925)}{483.6\times10^{-6}} = 448.562\ MPa.$$ ◄

Unloading $$\delta' = \frac{PL}{EA} = \frac{(436000)(0.355)}{(200\times10^{9})(1118.6\times10^{-6})} = 0.692\times10^{-3}\ m = 0.692\ mm. = 0.7\ mm$$

(c) $$\delta_p = \delta_m - \delta' = 0.796 - 0.692 = 0.104\ mm$$ ◄

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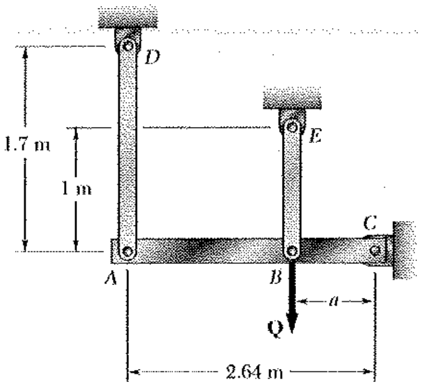
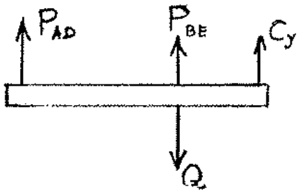
Problem 2.113



2.113 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5 × 6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

Statics: $\Sigma M_C = 0: \ 0.640(Q - P_{BE}) - 2.64\, P_{AD} = 0$

Deformation: $\delta_A = 2.64\,\theta, \quad \delta_B = a\theta = 0.640\,\theta$



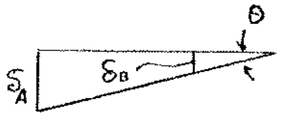
Elastic Analysis:

$A = (37.5)(6) = 225\text{ mm}^2 = 225\times10^{-6}\text{ m}^2$

$P_{AD} = \dfrac{EA}{L_{AD}}\delta_A = \dfrac{(200\times10^9)(225\times10^{-6})}{1.7}\delta_A = 26.47\times10^6\,\delta_A$

$= (26.47\times10^6)(2.64\,\theta) = 69.88\times10^6\,\theta$

$\sigma_{AD} = \dfrac{P_{AD}}{A} = 310.6\times10^9\,\theta$



$P_{BE} = \dfrac{EA}{L_{BE}}\delta_B = \dfrac{(200\times10^9)(225\times10^{-6})}{1.0}\delta_B = 45\times10^6\,\delta_B$

$= (45\times10^6)(0.640\,\theta) = 28.80\times10^6\,\theta$

$\sigma_{BE} = \dfrac{P_{BE}}{A} = 128\times10^9\,\theta$

From Statics, $Q = P_{BE} + \dfrac{2.64}{0.640}P_{AD} = P_{BE} + 4.125\,P_{AD}$

$= [28.80\times10^6 + (4.125)(69.88\times10^6)]\,\theta = 317.06\times10^6\,\theta$

θ_Y at yielding of link AD $\quad \sigma_{AD} = \sigma_Y = 250\times10^6 = 310.6\times10^9\,\theta$

$\theta_Y = 804.89\times10^{-6}$

$Q_Y = (317.06\times10^6)(804.89\times10^{-6}) = 255.2\times10^3\text{ N}$

(a) Since $Q = 260\times10^3 > Q_Y$, link AD yields. $\quad \sigma_{AD} = 250\text{ MPa}$ ◄

$P_{AD} = A\sigma_Y = (225\times10^{-6})(250\times10^{-6}) = 56.25\times10^3\text{ N}$

From Statics, $P_{BE} = Q - 4.125\,P_{AD} = 260\times10^3 - (4.125)(56.25\times10^3)$

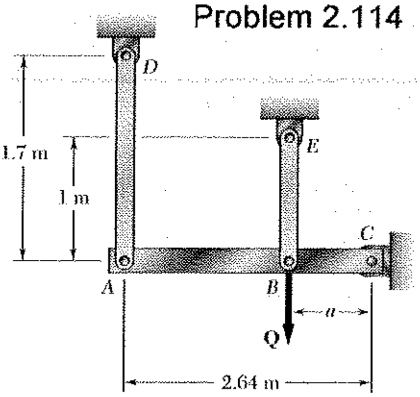
$P_{BE} = 27.97\times10^3\text{ N} \qquad \sigma_{BE} = \dfrac{P_{BE}}{A} = \dfrac{27.97\times10^3}{225\times10^{-6}} = 124.3\times10^6\text{ Pa}$

$= 124.3\text{ MPa}$ ◄

(b) $\delta_B = \dfrac{P_{BE}L_{BE}}{EA} = \dfrac{(27.97\times10^3)(1.0)}{(200\times10^9)(225\times10^{-6})} = 621.53\times10^{-6}\text{ m}$

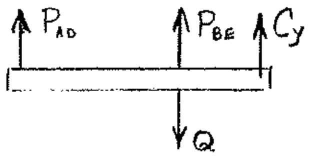
$\delta_B = 0.622\text{ mm}\downarrow$ ◄

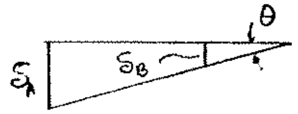
Problem 2.114



2.114 Solve Prob. 2.113, knowing that $a = 1.76$ m and that the magnitude of the force **Q** applied at B is gradually increased from zero to 135 kN.

2.113 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point B.





Statics: $\Sigma M_c = 0$: $1.76(Q - P_{BE}) - 2.64\,P_{AD} = 0$

Deformation: $\delta_A = 2.64\,\theta$, $\delta_B = 1.78\,\theta$

Elastic Analysis:

$A = (37.5)(6) = 225 \text{ mm}^2 = 225\times10^{-6} \text{ m}^2$

$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200\times10^9)(225\times10^{-6})}{1.7}\delta_A = 26.47\times10^6\,\delta_A$

$= (26.47\times10^6)(2.64\,\theta) = 69.88\times10^6\,\theta$

$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6\times10^9\,\theta$

$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200\times10^9)(225\times10^{-6})}{1.0}\delta_B = 45\times10^6\,\delta_B = (45\times10^6)(1.76\,\theta)$

$= 79.2\times10^6\,\theta$ $\qquad \sigma_{BE} = \frac{P_{BE}}{A} = 352\times10^9\,\theta$

From Statics, $Q = P_{BE} + \frac{2.64}{1.76}P_{AD} = P_{BE} + 1.500\,P_{AD}$

$= [73.8\times10^6 + (1.500)(69.88\times10^6)]\,\theta = 178.62\times10^6\,\theta$

θ_Y at yielding of link BE $\qquad \sigma_{BE} = \sigma_Y = 250\times10^6 = 352\times10^9\,\theta_Y$

$\theta_Y = 710.23\times10^{-6}$

$Q_Y = (178.62\times10^6)(710.23\times10^{-6}) = 126.86\times10^3$ N

Since $Q = 135\times10^3\text{N} > Q_Y$, link BE yields $\sigma_{BE} = \sigma_Y = 250$ MPa ◄

$P_{BE} = A\,\sigma_Y = (225\times10^{-6})(250\times10^6) = 56.25\times10^3$ N

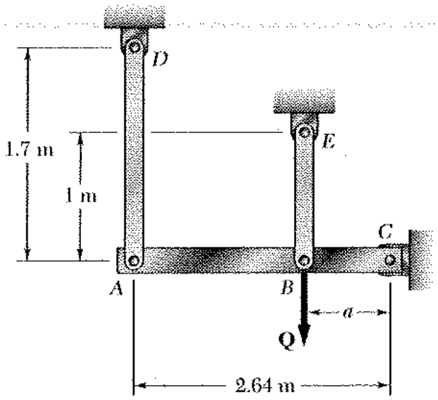
From Statics, $P_{AD} = \frac{1}{1.500}(Q - P_{BE}) = 52.5\times10^3$ N

$\sigma_{AD} = \frac{P_{AD}}{A} = \frac{52.5\times10^3}{225\times10^{-6}} = 233.3\times10^6 = 233$ MPa ◄

From elastic analysis of AD, $\theta = \frac{P_{AD}}{69.88\times10^6} = 751.29\times10^{-3}$ rad

$\delta_B = 1.76\,\theta = 1.322\times10^{-3}$ m $\qquad \delta_B = 1.322$ mm ↓ ◄

Problem 2.115



***2.115** Solve Prob. 2.113, assuming that the magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (*a*) the residual stress in each link, (*b*) the final deflection of point *B*. Assume that the links are braced so that they can carry compressive forces without buckling.

2.113 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5 × 6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

See solution to Problem 2.113 for the normal stresses in each link and the deflection of point B after loading

$\sigma_{AD} = 250 \times 10^6$ Pa $\qquad \sigma_{BE} = 124.3 \times 10^6$ Pa

$\delta_B = 621.53 \times 10^{-6}$ m

The elastic analysis given in the solution to PROBLEM 2.113 applies to the unloading

$Q' = 317.06 \times 10^6\ \theta'$

$$\theta' = \frac{Q}{317.06 \times 10^6} = \frac{260 \times 10^3}{317.06 \times 10^6} = 820.03 \times 10^{-6}$$

$\sigma'_{AD} = 310.6 \times 10^9\ \theta = (310.6 \times 10^9)(820.03 \times 10^{-6}) = 254.70 \times 10^6$ Pa

$\sigma'_{BE} = 128 \times 10^9\ \theta = (128 \times 10^9)(820.03 \times 10^{-6}) = 104.96 \times 10^6$ Pa

$\delta'_B = 0.640\ \theta' = 524.82 \times 10^{-6}$ m

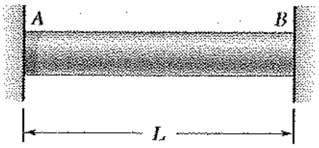
(a) Residual stresses

$\sigma_{AD,res} = \sigma_{AD} - \sigma'_{AD} = 250 \times 10^6 - 254.70 \times 10^{-6} = -4.70 \times 10^6$ Pa
$= -4.70$ MPa ◄

$\sigma_{BE,res} = \sigma_{BE} - \sigma'_{BE} = 124.3 \times 10^6 - 104.96 \times 10^6 = 19.34 \times 10^6$ Pa
$= 19.34$ MPa ◄

(b) $\delta_{B,P} = \delta_B - \delta'_B = 621.53 \times 10^{-6} - 524.82 \times 10^{-6}$
$= 96.71 \times 10^{-6}$ m $= 0.0967$ mm ↓ ◄

Problem 2.116



2.116 A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of 7°C. The steel is assumed to be elastoplastic with $\sigma_Y = 250$ MPa and $E = 200$GPa. Knowing that $\alpha = 11.7 \times 10^{-6}/°\mathrm{C}$, determine the stress in the bar (*a*) when the temperature is raised to 160°C, (*b*) after the temperature has returned to 7°C.

Let P be the compressive force in the rod.

Determine temperature change to cause yielding.

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_Y L}{E} + L\alpha(\Delta T)_Y = 0$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E\alpha} = \frac{250\times 10^6}{(200\times 10^9)(11.7\times 10^{-6})} = -106.8\ ^\circ C$$

But $\Delta T = 160 - 7 = 153\ ^\circ C > (\Delta T_Y)$

(a) Yielding occurs $\quad \sigma = -\sigma_Y = 250\,MPa$ ◄

Cooling: $(\Delta T)' = 153^\circ C.$

$$\delta' = \delta_P' + \delta_T' = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)' =$$

$$= -(200\times 10^9)(11.7\times 10^{-6})(153) = -358\ MPa.$$

(b) Residual stress

$$\sigma_{res} = -\sigma_Y - \sigma' = -250\times 10^6 + 358\times 10^6 = 108\ MPa$$

108 MPa ◄

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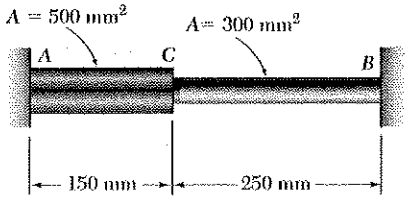
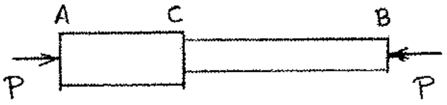
Problem 2.117





2.117 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25°C. The steel is assumed elastoplastic, with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The temperature of both portions of the rod is then raised to 150 °C. Knowing that $\alpha = 11.7 \times 10^{-6}/°\text{C}$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.

$A_{AC} = 500\times10^{-6}\ \text{m}^2 \qquad L_{AC} = 0.150\ \text{m}$

$A_{CB} = 300\times10^{-6}\ \text{m}^2 \qquad L_{CB} = 0.250\ \text{m}$

Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion CB.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\,\alpha(\Delta T)$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding, $P = P_Y = A_{CB}\sigma_Y = (300\times10^{-6})(250\times10^{6}) = 75\times10^{3}\ \text{N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

$$= \frac{75\times10^{3}}{(0.400)(200\times10^{9})(11.7\times10^{-6})}\left(\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}\right) = 90.812\ °\text{C}$$

Actual ΔT: $150\ °\text{C} - 25\ °\text{C} = 125\ °\text{C} > (\Delta T)_Y$ Yielding occurs.

For $\Delta T > (\Delta T)_Y \qquad P = P_Y = 75\times10^{3}\ \text{N}$

(a) $\sigma_{AC} = -\dfrac{P_Y}{A_{AC}} = -\dfrac{75\times10^{3}}{500\times10^{-6}} = -150\times10^{-6}\ \text{Pa}$ $\qquad \sigma_{AC} = -150\ \text{MPa}$ ◀

$\sigma_{CB} = -\dfrac{P_Y}{A_{CB}} = = -\sigma_Y$ $\qquad \sigma_{CB} = -250\ \text{MPa}$ ◀

(b) For $\Delta T > (\Delta T)_Y$, portion AC remains elastic.

$$\delta_{C/A} = -\frac{P_Y L_{AC}}{EA_{AC}} + L_{AC}\,\alpha(\Delta T)$$

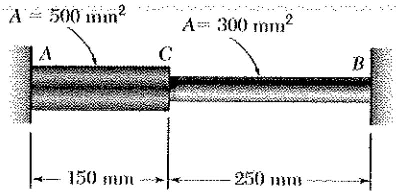
$$= -\frac{(75\times10^{3})(0.150)}{(200\times10^{9})(500\times10^{-6})} + (0.150)(11.7\times10^{-6})(125) = 106.9\times10^{-6}\ \text{m}$$

Since point A is stationary, $\delta_C = \delta_{C/A} = 106.9\times10^{-6}\ \text{m}$

$\delta_C = 0.1069\ \text{mm} \rightarrow$ ◀

Problem 2.118

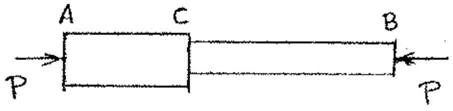
***2.118** Solve Prob. 2.117, assuming that the temperature of the rod is raised to 150 °C and then returned to 25°C.



2.117 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25°C. The steel is assumed elastoplastic, with *E* = 200 GPa and $\sigma_Y = 250$ MPa. The temperature of both portions of the rod is then raised to 150 °C. Knowing that $\alpha = 11.7\times10^{-6}/^\circ\text{C}$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.

$A_{AC} = 500\times10^{-6}\ \text{m}^2$ $\qquad L_{AC} = 0.150$ m

$A_{CB} = 300\times10^{-6}\ \text{m}^2$ $\qquad L_{CB} = 0.250$ m



Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion CB.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\alpha(\Delta T)$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

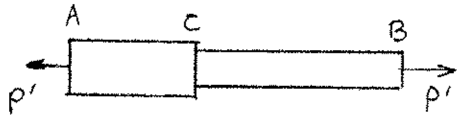
At yieding, $P = P_Y = A_{CB}\sigma_Y = (300\times10^{-6})(250\times10^{6}) = 75\times10^{3}$ N

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

$$= \frac{75\times10^{3}}{(0.400)(200\times10^{9})(11.7\times10^{-6})}\left(\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}\right) = 90.812\ ^\circ\text{C}$$

Actual ΔT: $150\ ^\circ\text{C} - 25\ ^\circ\text{C} = 125\ ^\circ\text{C} > (\Delta T)_Y$ $\qquad$ Yielding occurs.

For $\Delta T > (\Delta T)_Y$ $\qquad P = P_Y = 75\times10^{3}$ N



Cooling: $(\Delta T)' = 125\ ^\circ\text{C}$

$$P' = \frac{EL_{AB}\alpha(\Delta T)'}{\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)}$$

$$P' = \frac{(200\times10^{9})(0.400)(11.7\times10^{-6})(125)}{\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}} = 103.235\times10^{3}\ \text{N}$$

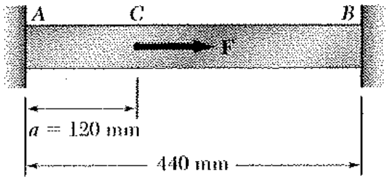
Residual force: $P_{res} = P' - P_Y = 103.235\times10^{3} - 75\times10^{3} = 28.235\times10^{3}$ N (tension)

(a) Residual stresses. $\quad \sigma_{AC} = \frac{P_{res}}{A_{AC}} = \frac{28.235\times10^{3}}{500\times10^{-6}}$ $\qquad \sigma_{AC} = 56.5$ MPa ◀

$\sigma_{CB} = \frac{P_{res}}{A_{CB}} = \frac{28.235\times10^{3}}{300\times10^{-6}}$ $\qquad \sigma_{CB} = 94.1$ MPa ◀

(b) Permanent deflection of point C. $\quad \delta_C = \frac{P_{res}L_{AC}}{EA_{AC}}$ $\qquad \delta_C = 0.0424$ mm → ◀

Problem 2.119

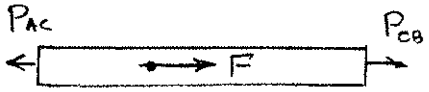


***2.119** Bar *AB* has a cross-sectional area of 1200 mm² and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point *C*, (*b*) the residual stress in the bar.

$$A = 1200\ \text{mm}^2 = 1200\times10^{-6}\ \text{m}^2$$

Force to yield portion AC : $P_{AC} = A\sigma_Y = (1200\times10^{-6})(250\times10^{6})$

$$= 300\times10^3\ \text{N}$$



For equilibrium, $F + P_{CB} - P_{AC} = 0$

$$P_{CB} = P_{AC} - F = 300\times10^3 - 520\times10^3$$

$$= -220\times10^3\ \text{N}$$

$$\delta_C = -\frac{P_{CB}L_{CB}}{EA} = \frac{(220\times10^3)(0.440-0.120)}{(200\times10^9)(1200\times10^{-6})} = 0.293333\times10^{-3}\ \text{m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220\times10^3}{1200\times10^{-6}} = -183.333\times10^6\ \text{Pa}$$

Unloading.

$$\delta_C' = \frac{P_{AC}'L_{AC}}{EA} = -\frac{P_{CB}'L_{CB}}{EA} = \frac{(F-P_{AC}')L_{CB}}{EA}$$

$$P_{AC}'\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{FL_{CB}}{EA}$$

$$P_{AC}' = \frac{FL_{CB}}{L_{AC}+L_{CB}} = \frac{(520\times10^3)(0.440-0.120)}{0.440} = 378.182\times10^3\ \text{N}$$

$$P_{CB}' = P_{AC}' - F = 378.182\times10^3 - 520\times10^3 = -141.818\times10^3\ \text{N}$$

$$\sigma_{AC}' = \frac{P_{AC}'}{A} = \frac{378.182\times10^3}{1200\times10^{-6}} = 315.152\times10^6\ \text{Pa}$$

$$\sigma_{BC}' = \frac{P_{BC}'}{A} = -\frac{141.818\times10^3}{1200\times10^{-6}} = -118.182\times10^6\ \text{Pa}$$

$$\delta_C' = \frac{(378.182)(0.120)}{(200\times10^9)(1200\times10^6)} = 0.189091\times10^{-3}\ \text{m}$$

(a) $\delta_{Cp} = \delta_C - \delta_C' = 0.293333\times10^{-3} - 0.189091\times10^{-3} = 0.1042\times10^{-3}\ \text{m}$

$= 0.1042\ \text{mm}$ ◀

(b) $\sigma_{AC,res} = \sigma_Y - \sigma_{AC}' = 250\times10^6 - 315.152\times10^6 = -65.2\times10^6\ \text{Pa}$

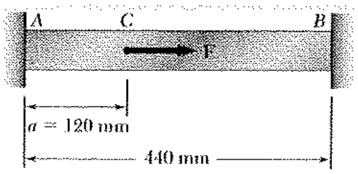
$= -65.2\ \text{MPa}$ ◀

$\sigma_{CB,res} = \sigma_{CB} - \sigma_{CB}' = -183.333\times10^6 + 118.182\times10^6 = -65.2\times10^6\ \text{Pa}$

$= -65.2\ \text{MPa}$ ◀

Problem 2.120

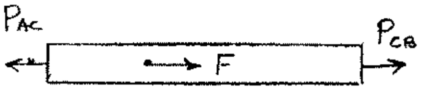
***2.120** Solve Prob. 2.119, assuming that $a = 180$ mm.



***2.119** Bar *AB* has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point *C*, (*b*) the residual stress in the bar.

$$A = 1200\ \text{mm}^2 = 1200\times10^{-6}\ \text{m}^2$$

Force to yield portion AC: $P_{AC} = A\sigma_Y = (1200\times10^{-6})(250\times10^{6}) = 300\times10^{3}\ \text{N}$



For equilibrium $F + P_{CB} - P_{AC} = 0$

$$P_{CB} = P_{AC} - F = 300\times10^3 - 520\times10^3 = -220\times10^3\ \text{N}$$

$$\delta_C = -\frac{P_{CB}L_{CB}}{EA} = \frac{(220\times10^3)(0.440-0.180)}{(200\times10^9)(1200\times10^{-6})} = 0.238333\times10^{-3}\ \text{m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = -\frac{220\times10^3}{1200\times10^{-6}} = -183.333\times10^6\ \text{Pa}$$

Unloading

$$\delta_C' = \frac{P_{AC}'L_{AC}}{EA} = -\frac{P_{CB}'L_{CB}}{EA} = \frac{(F-P_{AC}')L_{CB}}{EA} \quad \therefore \quad P_{AC}'\left(\frac{L_{AC}}{EA}+\frac{L_{CB}}{EA}\right) = \frac{FL_{CB}}{EA}$$

$$P_{AC}' = \frac{FL_{CB}}{L_{AC}+L_{CB}} = \frac{(520\times10^3)(0.440-0.180)}{0.440} = 307.273\times10^3\ \text{N}$$

$$P_{CB}' = P_{AC}' - F = 307.273\times10^3 - 520\times10^3 = -212.727\times10^3\ \text{N}$$

$$\delta_C' = \frac{(307.273\times10^3)(0.180)}{(200\times10^9)(1200\times10^{-6})} = 0.230455\times10^{-3}\ \text{m}$$

$$\sigma_{AC}' = \frac{P_{AC}'}{A} = \frac{307.273\times10^3}{1200\times10^{-6}} = 256.061\times10^6\ \text{Pa}$$

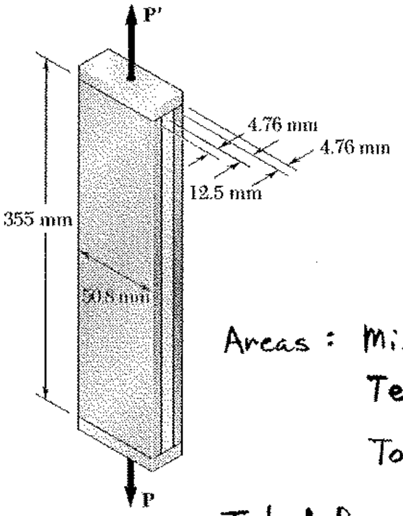
$$\sigma_{CB}' = \frac{P_{CB}'}{A} = \frac{-212.727\times10^3}{1200\times10^{-6}} = -177.273\times10^6\ \text{P}$$

(a) $\delta_{cp} = \delta_c - \delta_c' = 0.238333\times10^{-3} - 0.230455\times10^{-3} = 0.00788\times10^{-3}\ \text{m}$
$= 0.00788\ \text{mm}$

(b) $\sigma_{AC,res} = \sigma_{AC} - \sigma_{AC}' = 250\times10^6 - 256.061\times10^6 = -6.06\times10^6\ \text{Pa}$
$= -6.06\ \text{MPa}$

$\sigma_{CB,res} = \sigma_{CB} - \sigma_{CB}' = -183.333\times10^6 + 177.273\times10^6 = -6.06\times10^6\ \text{Pa}$
$= -6.06\ \text{MPa}$

Problem 2.121



***2.121** For the composite bar of Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero to 436 kN and then decreased back to zero.

2.111 Two tempered-steel bars, each 4.76 mm thick, are bonded to a 12.5-mm mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 200$ GPa and with yield strengths equal to 690 MPa and 345 MPa, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 1.016$ mm and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

Areas : Mild steel $A_1 = (0.0125)(0.0508) = 635\times10^{-6}\ m^2$

Tempered steel $A_2 = (2)(0.00476)(0.0508) = 483.6\times10^{-6}\ m^2$

Total: $A = A_1 + A_2 = 1118.6\times10^{-6}\ m^2$

Total force to yield the mild steel

$$\sigma_{Y_1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y_1} = (1118.6\times10^{-6})(345\times10^{6}) = 385.917\text{kN}.$$

$P > P_Y$; therefore mild steel yields

Let P_1 = force carried by mild steel
P_2 = force carried by tempered steel

$$P_1 = A_1\sigma_{Y_1} = (635\times10^{-6})(345\times10^{6}) = 219.075\ \text{kN}$$

$$P_1 + P_2 = P, \qquad P_2 = P - P_1 = 436\times10^{3} - 219.075\times10^{3} = 217\ \text{kN}.$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{217\times10^{3}}{483.6\times10^{-6}} = 448.7\ \text{MPa}.$$

Unloading $$\sigma' = \frac{P}{A} = \frac{436\times10^{3}}{1118.6\times10^{-6}} = 389.8\ \text{MPa}.$$

Residual stresses

mild steel $\sigma_{1,res} = \sigma_1 - \sigma' = 345\text{MPa} - 389.8\text{MPa} = -44.8\ \text{MPa}.$

tempered steel $\sigma_{2,res} = \sigma_2 - \sigma_1 = 448.7 - 389.8$

$= 58.9\ \text{MPa}.$ ◄

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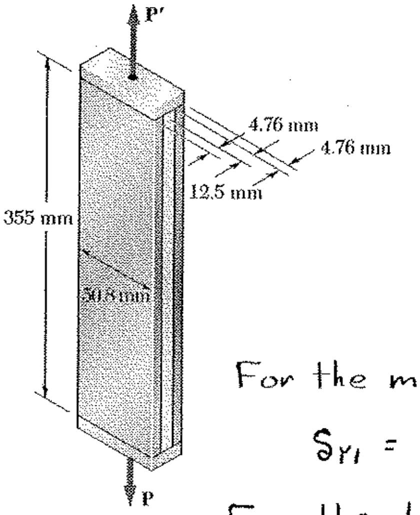
Problem 2.122



***2.122** For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if *P* is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 10$ mm and is then decreased back to zero.

2.111 Two tempered-steel bars, each 4.76 mm thick, are bonded to a 12.5-mm mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude *P*. Both steels are elastoplastic with $E = 200$ GPa and with yield strengths equal to 690 MPa and 345 MPa, respectively, for the tempered and mild steel. The load *P* is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 1.016$ mm and then decreased back to zero. Determine (*a*) the maximum value of *P*, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

For the mild steel $A_1 = (0.0125)(0.0508) = 635 \times 10^{-6}\ m^2$.

$$\delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(0.355\ (345\times10^6)}{200\times10^9} = 0.612\times10^{-3}\ m$$

For the tempered steel $A_2 = 2(0.00476)(0.0508) = 483.6\times10^{-6}\ m^2$

$$\delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(0.355)(690\times10^6)}{200\times10^9} = 1.225\times10^{-3}\ m$$

Total area: $A = A_1 + A_2 = 1118.6\times10^{-6}\ m^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$ The mild steel yields. Tempered steel is elastic.

Forces $P_1 = A_1\sigma_{Y1} = (635\times10^{-6})(345\times10^6) = 219.075$ kN

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(200\times10^9)(483.6\times10^{-6})(0.001016)}{0.355} = 276.81\ kN$$

Stresses $\sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 345$ MPa

$$\sigma_2 = \frac{P_2}{A_2} = \frac{276.81\times10^3}{483.6\times10^{-6}} = 572.4\ MPa$$

Unloading $\sigma' = \frac{P}{A} = \frac{495900}{1118.6\times10^{-6}} = 443.3$ MPa.

Residual stresses

$$\sigma_{1,res} = \sigma_1 - \sigma' = 345 - 443.3 = -98.3\ MPa$$

$$\sigma_{2,res} = \sigma_2 - \sigma' = 572.4 - 443.3 = 129.1\ MPa$$ ◄

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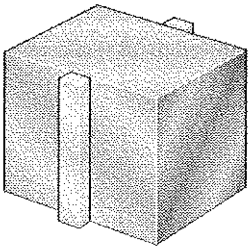
Problem 2.123



***2.123** A narrow bar of aluminum is bonded to the side of a thick steel plate as shown. Initially, at $T_1 = 21°\text{C}$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1, determine (*a*) the highest temperature T_2 that does *not* result in residual stresses, (*b*) the temperature T_2 that will result in a residual stress in the aluminum equal to 400 MPa. Assume $\alpha_a = 23 \times 10^{-6}/°\text{C}$ for the aluminum and $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 75$ GPa and $\sigma_Y = 400$ MPa. (*Hint:* Neglect the small stresses in the plate.)

Determine temperature change to cause yielding

$$\delta = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T_Y) = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{400 \times 10^6}{(75 \times 10^9)(23 - 11.7)(10^{-6})} = 472°\text{C}$$

(a) $T_{2Y} = T_1 + (\Delta T)_Y = 21 + 472 = 493°\text{C}$

After yielding

$$\delta = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling

$$\delta' = \frac{P'L}{AE} + L\alpha_a(\Delta T)' = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{res} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

Set $\sigma_{res} = -\sigma_Y$

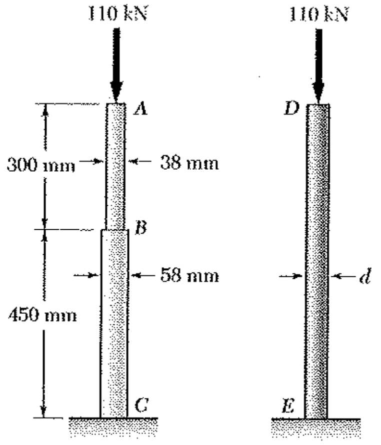
$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\Delta T = \frac{2\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{400 \times 10^6}{(75 \times 10^9)(23 - 11.7)(10^{-6})} = 944°\text{C}$$

(b) $T_2 = T_1 + \Delta T = 21 + 944 = 965°\text{C}$

If $T_2 > 965°\text{C}$ the aluminum bar will most likely yield in compression.

Problem 2.124



2.124 The aluminum rod *ABC* ($E = 70$ GPa), which consists of two cylindrical portions *AB* and *BC*, is to be replaced with a cylindrical steel rod *DE* ($E = 200$ GPa) of the same overall length. Determine the minimum required diameter *d* of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 165 MPa.

Deformation of aluminum rod

$$\delta_A = \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E} = \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$= \frac{110\times10^3}{70\times10^9}\left(\frac{0.3}{\frac{\pi}{4}(0.038)^2} + \frac{0.45}{\frac{\pi}{4}(0.058)^2}\right) = 0.683\times10^{-3}\ \text{m.}$$

Steel rod $\quad \delta = 0.683\times10^{-3}\ \text{m.}$

$$\delta = \frac{PL}{EA} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(110\times10^3)(0.75)}{(200\times10^9)(0.683\times10^{-3})} = 604\times10^{-6}\ \text{m}^2$$

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{110\times10^3}{165\times10^6} = 0.667\times10^{-3}\ \text{m}^2$$

Required area is the larger value $\quad A = 0.667\times10^{-3}\ \text{m}^2.$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.667\times10^{-3})}{\pi}} = 0.029\text{m} = 29\text{mm}$$ ◀

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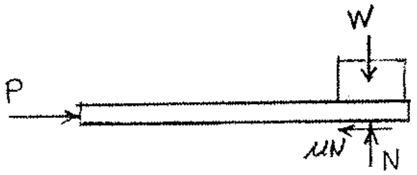
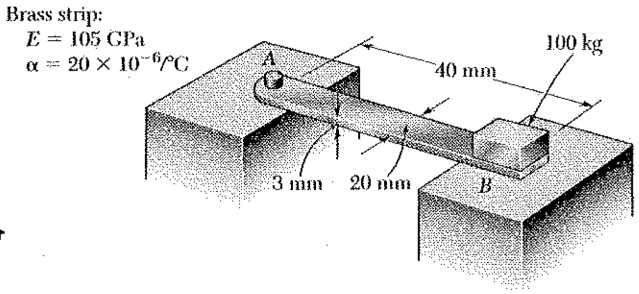
Problem 2.125

2.125 The brass strip *AB* has been attached to a fixed support at *A* and rests on a rough support at *B*. Knowing that the coefficient of friction is 0.60 between the strip and the support at *B*, determine the decrease in temperature for which slipping will impend.





$+\uparrow \Sigma F_y = 0: \quad N - W = 0 \qquad N = W$

$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad P - \mu N = 0 \qquad P = \mu W = \mu m g$

$\delta = -\frac{PL}{EA} + L\alpha(\Delta T) = 0 \qquad \Delta T = \frac{P}{EA\alpha} = \frac{\mu m g}{EA\alpha}$

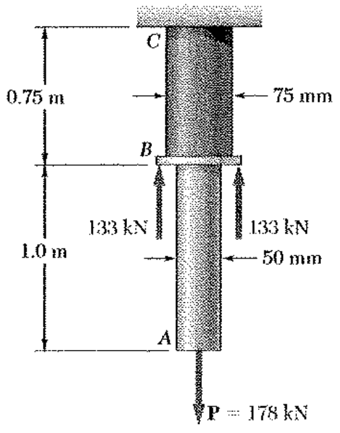
Data: $\mu = 0.60 \qquad A = (20)(3) = 60\ mm^2 = 60 \times 10^{-6}\ m^2$

$m = 100\ kg \qquad g = 9.81\ m/s^2 \qquad E = 105 \times 10^9\ Pa$

$\Delta T = \frac{(0.60)(100)(9.81)}{(105\times 10^9)(60\times 10^{-6})(20\times 10^{-6})} = 4.67\,°C$ ◄

Problem 2.126

2.126 Two solid cylindrical rods are joined at *B* and loaded as shown. Rod *AB* is made of steel (*E* = 200 GPa), and rod *BC* of brass (*E* = 105 GPa). Determine (*a*) the total deformation of the composite rod *ABC*, (*b*) the deflection of point *B*.



Portion AB: $P_{AB} = 178\ kN \qquad L_{AB} = 1m \qquad d = 50\ mm.$

$A_{AB} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.05)^2 = 0.001963\ m^2 \qquad E_{AB} = 200\ GPa.$

$\delta_{AB} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} = \frac{(178\times 10^3)(1)}{(200\times 10^9)(0.001963)} = 0.45m \times 10^{-3}\ m$

Portion BC: $P_{BC} = -88\ kN \qquad L_{BC} = 0.75m.,\ d = 75\ mm$

$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.075)^2 = 0.00442\ m^2 \qquad E_{BC} = 105\ GPa$

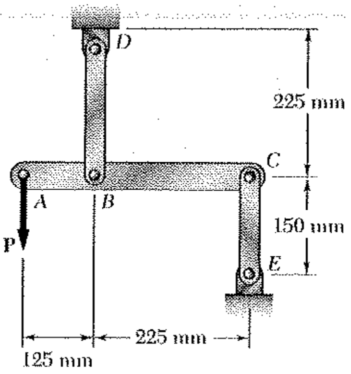
$\delta_{BC} = \frac{P_{BC} L_{BC}}{E_{BC} A_{BC}} = \frac{(-88\times 10^3)(0.75)}{(105\times 10^9)(0.00442)} = -0.142\times 10^{-3}\ m$

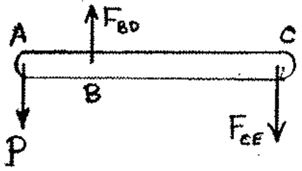
(a) $\delta = \delta_{AB} + \delta_{BC} = 0.45 - 0.142 = 0.308\ mm \qquad \delta = 0.308\ mm$ ◄

(b) $\delta_B = -\delta_{BC} \qquad \delta_B = 0.142\ mm$ ◄

Problem 2.127

2.127 Link *BD* is made of brass (E = 105 GPa) and has a cross-sectional area of 250 mm^2. Link *CE* is made of aluminum (E = 72 GPa) and has a cross-sectional area of 450 mm^2. Determine the maximum force **P** that can be applied vertically at point *A* if the deflection of *A* is not to exceed 0.35 mm.





Use member ABC as a free body

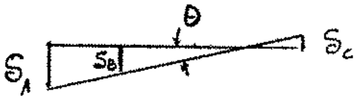
$$\circlearrowleft\!+\ \Sigma M_C = 0,\quad 350P - 225F_{BD} = 0,\qquad F_{BD} = 1.5556\ P$$

$$\circlearrowleft\!+\ \Sigma M_B = 0\quad 125P\ \ 225F_{CE} = 0,\qquad F_{CE} = 0.5556\ P$$

$$\delta_B = \delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}} = \frac{(1.5556P)(0.225)}{(105\times10^{9})(250\times10^{-6})} = 13.3334\times10^{-9}\ P \downarrow$$

$$\delta_C = \delta_{CE} = \frac{F_{CE}L_{CE}}{E_{CE}A_{CE}} = \frac{(0.5556)(0.15)}{(72\times10^{9})(450\times10^{-6})} = 2.5722\times10^{-9}\ P \uparrow$$

From the deformation diagram



Deformation Diagram

$$\text{Slope } \theta = \frac{\delta_B + \delta_C}{l_{BC}} = \frac{15.9056\times10^{-9}\ P}{0.225}$$

$$= 70.6916\times10^{-9}\ P$$

$$\delta_A = \delta_B + l_{AB}\,\theta$$

$$= 13.3334\times10^{-9}\ P + (0.125)(70.69\times10^{-9})\ P$$

$$= 22.17\times10^{-9}\ P$$

Apply displacement limit $\delta_A = 0.00035 = 22.17\times10^{-9}\ P$.

$$P = \frac{0.00035}{22.17\times10^{-9}} = 15.787\ \text{kN}. = 15.79\ \text{kN}$$

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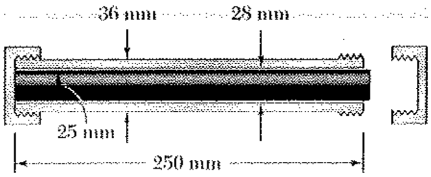
Problem 2.128



2.128 A 250-mm-long aluminum tube ($E = 70$ GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105$ GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

$$A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{rod} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$\delta_{tube} = \frac{PL}{E_{tube}A_{tube}} = \frac{P(0.250)}{(70\times10^9)(402.12\times10^{-6})} = 8.8815\times10^{-9} P$$

$$\delta_{rod} = -\frac{PL}{E_{rod}A_{rod}} = \frac{P(0.250)}{(105\times10^6)(490.87\times10^{-6})} = -4.8505\times10^{-9} P$$

$$\delta^* = (\tfrac{1}{4}\text{ turn}) \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375\times10^{-6} \text{ m}$$

$$\delta_{tube} = \delta^* + \delta_{rod} \quad \text{or} \quad \delta_{tube} - \delta_{rod} = \delta^*$$

$$8.8815\times10^{-9} P + 4.8505\times10^{-9} P = 375 \times10^{-6}$$

$$P = \frac{0.375\times10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308\times10^{3} \text{ N}$$

(a) $$\sigma_{tube} = \frac{P}{A_{tube}} = \frac{27.308\cdot10^3}{402.12\times10^{-6}} = 67.9\times10^6 \text{ Pa} = 67.9 \text{ MPa}$$

$$\sigma_{rod} = -\frac{P}{A_{rod}} = -\frac{27.308\times10^3}{490.87\times10^{-6}} = -55.6\times10^6 \text{ Pa} = -55.6 \text{ MPa}$$

(b) $$\delta_{tube} = (8.8815\times10^{-9})(27.308\times10^3) = 242.5\times10^{-6} \text{ m} = 0.2425 \text{ mm}$$

$$\delta_{rod} = -(4.8505\times10^{-9})(27.308\times10^3) = -132.5\times10^{-6} \text{ m} = -0.1325 \text{ mm}$$

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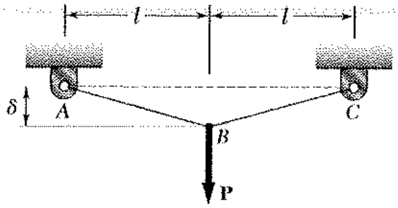
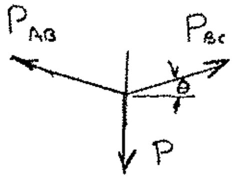
Problem 2.129



2.129 The uniform wire *ABC*, of unstretched length $2l$, is attached to the supports shown and a vertical load **P** is applied at the midpoint *B*. Denoting by *A* the cross-sectional area of the wire and by *E* the modulus of elasticity, show that, for $\delta << l$, the deflection at the midpoint *B* is

$$\delta = l\sqrt[3]{\frac{P}{AE}}$$



Use approximation,

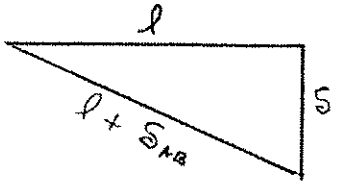
$$\sin\theta \approx \tan\theta \approx \frac{\delta}{l}$$

Statics $+\uparrow \Sigma F_y = 0: \quad 2P_{AB}\sin\theta - P = 0$

$$P_{AB} = \frac{P}{2\sin\theta} \approx \frac{Pl}{2\delta}$$

Elongation. $\delta_{AB} = \frac{P_{AB}l}{AE} = \frac{Pl^2}{2AE\delta}$

Deflection



From the right triangle

$$(l + \delta_{AB})^2 = l^2 + \delta^2$$

$$\delta^2 = \cancel{l^2} + 2l\delta_{AB} + \delta_{AB}^2 - \cancel{l^2}$$

$$= 2l\delta_{AB}\left(1 + \frac{1}{2}\frac{\delta_{AB}}{l}\right) \approx 2l\delta_{AB}$$

$$\approx \frac{Pl^3}{AE\delta}$$

$$\delta^3 \approx \frac{Pl^3}{AE} \quad \therefore \delta \approx l\sqrt[3]{\frac{P}{AE}}$$ ◄

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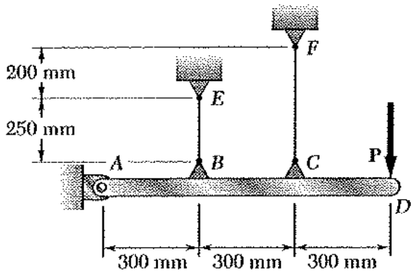
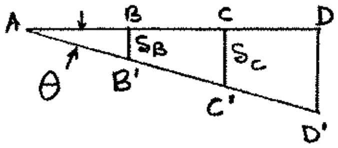
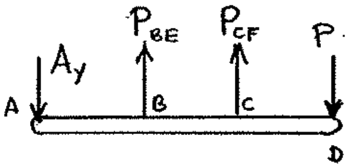
Problem 2.130

2.130 The rigid bar *AD* is supported by two steel wires of 1.5 mm diameter ($E = 200$ GPa) and a pin and bracket at *D*. Knowing that the wires were initially taut, determine (*a*) the additional tension in each wire when a 900 N load **P** is applied at *D*, (*b*) the corresponding deflection of point *D*.







Let θ be the rotation of bar ABCD

Then $\delta_B = 0.3\,\theta$

$\delta_C = 0.6\,\theta$

$$\delta_B = \frac{P_{BE}L_{BE}}{AE}$$

$$P_{BE} = \frac{EA\,\delta_{BE}}{L_{BE}} = \frac{(200\times10^9)\frac{\pi}{4}(0.0015)^2(0.3\theta)}{0.25}$$

$$= 424115\,\theta$$

$$\delta_C = \frac{P_{CF}L_{CF}}{EA}$$

$$P_{CF} = \frac{EA\,\delta_{CF}}{L_{CF}} = \frac{(200\times10^9)\frac{\pi}{4}(0.0015)^2(0.6)}{0.45}$$

$$= 471239\,\theta.$$

Using free body ABCD

$$\circlearrowleft\Sigma M_A = 0 \qquad 0.3P_{BE} + 0.6P_{CF} - 0.9P = 0$$

$$(0.3)(.424115\,\theta) + (0.6)(471239\,\theta) - (0.9)(900) = 0$$

$$409978\,\theta = 810$$

$$\theta = 1.976\times10^{-3}\text{ rad}$$

(a) $P_{BE} = (424115)(1.976\times10^{-3}) = 838\text{ N.}$

$P_{CF} = (471239)(1.976\times10^{-3}) = 931\text{ N.}$

(b) $\delta_D = 0.9\,\theta = (0.9)(1.976\times10^{-3}) = 1.78\times10^{-3}\text{ m}$

$= 1.78\text{ mm}$

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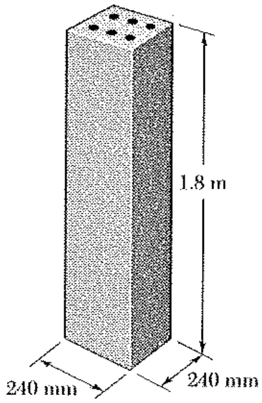
Problem 2.131

2.131 The concrete post ($E_c = 25$ GPa and $\alpha_c = 9.9 \times 10^{-6}/°\text{C}$) is reinforced with six steel bars, each of 22-mm diameter (E_s = 200 GPa and $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 35°C.



$$A_s = 6\,\frac{\pi}{4}d^2 = 6\cdot\frac{\pi}{4}(22)^2 = 2.2808\times10^3\,\text{mm}^2 = 2.2808\times10^{-3}\,\text{m}^2$$

$$A_c = 240^2 - A_s = 240^2 - 2.2808\times10^3 = 55.32\times10^3\,\text{mm}^2$$
$$= 55.32\times10^{-3}\,\text{m}^2$$

Let P_c = tensile force developed in the concrete.

For equilibrium with zero total force, the compressive force in the six steel rods equals P_c.

Strains: $\varepsilon_s = -\dfrac{P_c}{E_sA_s} + \alpha_s(\Delta T)$, $\varepsilon_c = \dfrac{P_c}{E_cA_c} + \alpha_c(\Delta T)$

Matching: $\varepsilon_c = \varepsilon_s$ $\dfrac{P_c}{E_cA_c} + \alpha_c(\Delta T) = -\dfrac{P_c}{E_sA_s} + \alpha_s(\Delta T)$

$$\left(\frac{1}{E_cA_c} + \frac{1}{E_sA_s}\right)P_c = (\alpha_s - \alpha_c)(\Delta T)$$

$$\left[\frac{1}{(25\times10^9)(55.32\times10^{-3})} + \frac{1}{(200\times10^9)(2.2808\times10^{-3})}\right]P_c = (1.8\times10^{-6})(35)$$

$$P_c = 21.61\times10^3\ \text{N}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{21.61\times10^3}{55.32\times10^{-3}} = 0.391\times10^6\ \text{Pa} = 0.391\ \text{MPa} \quad ◄$$

$$\sigma_s = -\frac{P_c}{A_c} = \frac{21.61\times10^3}{2.2808\times10^{-3}} = -9.47\times10^6\ \text{Pa} = -9.47\ \text{MPa} \quad ◄$$

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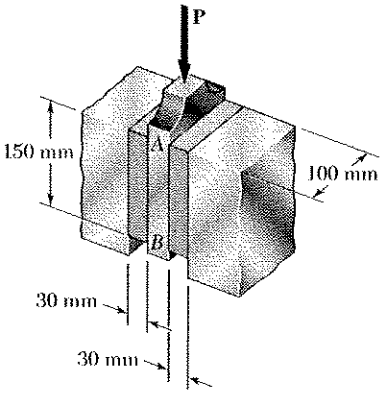
Problem 2.132

2.132 A vibration isolation unit consists of two blocks of hard rubber with a modulus of rigidity $G = 19$ MPa bonded to a plate AB and to rigid supports as shown. Denoting by P the magnitude of the force applied to the plate and by δ the corresponding deflection, determine the effective spring constant, $k = P/\delta$, of the system.



Shearing strain $\gamma = \frac{\delta}{h}$

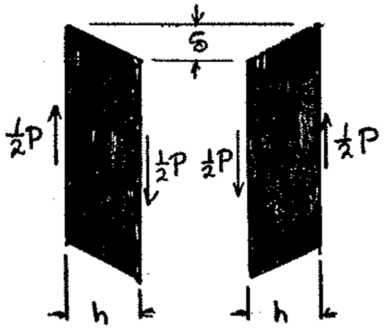
Shearing stress $\tau = G\gamma = \frac{G\delta}{h}$

Force $\frac{1}{2}P = A\tau = \frac{GA\delta}{h}$

$$P = \frac{2GA\delta}{h}$$

Effective spring constant

$$k = \frac{P}{\delta} = \frac{2GA}{h}$$



Data: $G = 19$ MPa

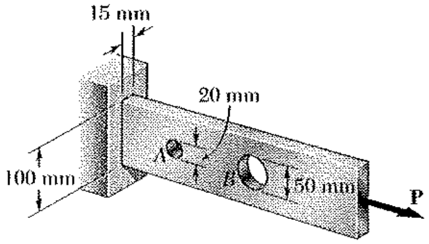
$A = (0.15)(0.1) = 0.015 \text{ m}^2$

$h = 30$ mm

$$k = \frac{2(19\times10^6)(0.015)}{0.03} = 19 \text{ N/m}.$$

Problem 2.133

2.133 Knowing that $\sigma_{all} = 120$ MPa, determine the maximum allowable value of the centric axial load **P**.



At hole A: $r = \frac{1}{2}(20) = 10$ mm

$d = D - 2r = 100 - 20 = 80$ mm

$A_{net} = dt = (80)(15) = 1200 \text{ mm}^2 = 1.20 \times 10^{-3} \text{ m}^2$

$\frac{r}{d} = \frac{10}{80} = 0.125$

From Fig. 2.64a, $K = 2.65$

$\sigma_{max} = \frac{KP}{A_{net}}$ $\quad P = \frac{A_{net}\sigma_{max}}{K} = \frac{(1.20\times10^{-3})(120\times10^6)}{2.65} = 54.3\times10^3$ N

At hole B: $r = \frac{1}{2}(50) = 25$ mm $\quad d = 100 - 50 = 50$ mm

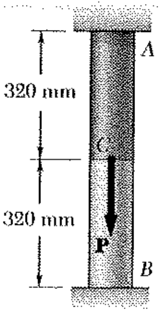
$A_{net} = (50)(15) = 750 \text{ mm}^2 = 750\times10^{-6} \text{ m}^2$, $\quad \frac{r}{d} = \frac{25}{50} = 0.50$, $K = 2.16$

$P = \frac{(750\times10^{-6})(120\times10^6)}{2.16} = 41.7\times10^3$ N

Allowable value of P is the smaller. $\quad P = 41.7\times10^3 \text{ N} = 41.7$ kN

Problem 2.134

2.134 Rod *AB* consists of two cylindrical portions *AC* and *BC*, each with a cross-sectional area of 2950 mm^2. Portion *AC* is made of a mild steel with E = 200 GPa and σ_Y = 250 MPa, and portion *CB* is made of a high-strength steel with E = 200 GPa and σ_Y = 345 MPa. A load **P** is applied at *C* as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of *C* if *P* is gradually increased from zero to 1625 kN and then reduced back to zero, (*b*) the permanent deflection of *C*.

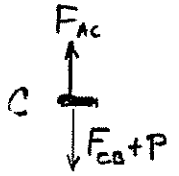


Displacement at C to cause yielding of AC

$$\delta_{C,Y} = L_{AC}\varepsilon_{Y,AC} = \frac{L_{AC}\sigma_{Y,AC}}{E} = \frac{(0.320)(250\times10^6)}{200\times10^9} = 0.400\times10^{-3}\ \text{m}$$

Corresponding force $F_{AC} = A\sigma_{Y,AC} = (2950\times10^{-6})(250\times10^6) = 737.5\times10^3$ N

$$F_{CB} = -\frac{EA\delta_C}{L_{CB}} = -\frac{(200\times10^9)(2950\times10^{-6})(0.400\times10^{-3})}{0.320} = -737.5\times10^3\ \text{N}$$



For equilibrium of element at C,

$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 1475\times10^3\ \text{N}$$

Since applied load $P = 1625\times10^3$ N $> 1475\times10^3$ N, portion AC yields.

$$F_{CB} = F_{AC} - P = 737.5\times10^3 - 1625\times10^3\ \text{N} = -887.5\times10^3\ \text{N}$$

(a) $$\delta_C = -\frac{F_{CB}L_{CB}}{EA} = \frac{(887.5\times10^3)(0.320)}{(200\times10^9)(2950\times10^{-6})} = 0.48136\times10^{-3}\ \text{m}$$

$$= 0.481\ \text{mm}\ \downarrow$$

Maximum stresses. $\sigma_{AC} = \sigma_{Y,AC} = 250$ MPa

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{887.5\times10^3}{2950\times10^{-6}} = -300.81\times10^6\ \text{Pa} = -301\ \text{MPa}$$

(b) Deflection and forces for unloading.

$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore \quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{CB}} = -P'_{AC}$$

$$P' = 1625\times10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \qquad P'_{AC} = 812.5\times10^3\ \text{N}$$

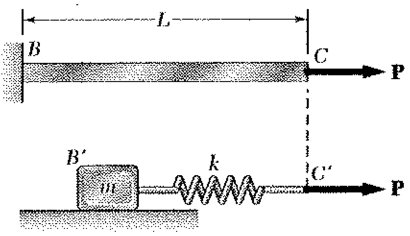
$$\delta' = \frac{(812.5\times10^3)(0.320)}{(200\times10^9)(2950\times10^{-6})} = 0.44068\times10^{3}\ \text{m}$$

$$\delta_p = \delta_m - \delta' = 0.48136\times10^{-3} - 0.44068\times10^{-3} = 0.04068\times10^{-3}\ \text{m}$$

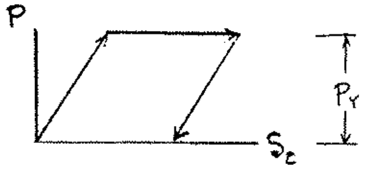
Permanent deflection of C $\qquad \delta_p = 0.0407\ \text{mm}\ \downarrow$

Problem 2.135

2.135 The uniform rod *BC* has a cross-sectional area *A* and is made of a mild steel which can be assumed to be elastoplastic with a modulus of elasticity *E* and a yield strength σ_y. Using the block-and-spring system shown, it is desired to simulate the deflection of end *C* of the rod as the axial force **P** is gradually applied and removed, that is, the deflection of points *C* and *C'* should be the same for all values of *P*. Denoting by μ the coefficient of friction between the block and the horizontal surface, derive an expression for (*a*) the required mass *m* of the block, (*b*) the required constant *k* of the spring.



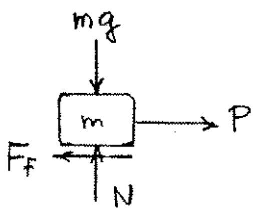
Force-deflection diagram for point C of rod BC.



For $P < P_Y = A\sigma_Y$

$$\delta_c = \frac{PL}{EA} \qquad P = \frac{EA}{L}\delta_c$$

$$P_{max} = P_Y = A\sigma_Y$$



Force-deflection diagram for point C' of block and spring system.

$+\uparrow \Sigma F_y = 0: \quad N - mg = 0 \qquad N = mg$

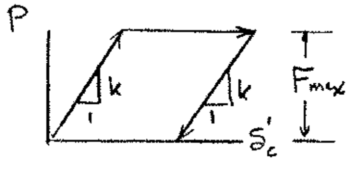
$\xrightarrow{+} \Sigma F_x = 0: \quad P - F_f = 0 \qquad P = F_f$

If block does not move, i.e $F_f < \mu N = \mu mg$ or $P < \mu mg$,

then $\delta_c' = \frac{P}{k}$ or $P = k\delta_c'$

If $P = \mu mg$, then slip at $P = F_m = \mu mg$ occurs.

If the force P is the removed, the spring returns to its initial length.



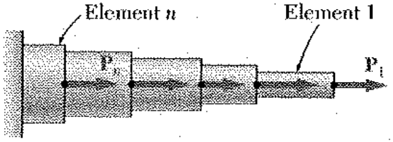
(a) Equating P_Y and F_{max},

$$A\sigma_Y = \mu mg \qquad m = \frac{A\sigma_Y}{\mu g}$$

(b) Equating slopes,

$$k = \frac{EA}{L}$$

PROBLEM 2.C1



2.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (*a*) Write a computer program that can be used to determine the average normal stress in each element, the deformation of each element, and the total deformation of the rod. (*b*) Use this program to solve Probs. 2.20 and 2.126.

SOLUTION

```
FOR EACH ELEMENT, ENTER
    $L_i$, $A_i$, $E_i$
COMPUTE DEFORMATION
    UPDATE AXIAL LOAD   $P = P + P_i$
    COMPUTE FOR EACH ELEMENT
        $\sigma_i = P/A_i$
        $\delta_i = PL_i/A_iE_i$
    TOTAL DEFORMATION:
        UPDATE THROUGH n ELEMENTS
            $\delta = \delta + \delta_i$
```

PROGRAM OUTPUT

```
Problem  2.20
  Element  Stress (MPa)  Deformation (mm)

     1       19.0986        .1091
     2      -12.7324       -.0909
  Total Deformation =     .0182 mm

Problem  2.126
  Element  Stress (MPa)  Deformation (mm)

     1        90.6775        0.4534
     2       -19.9095        0.1422
  Total Deformation =  0.3112
```

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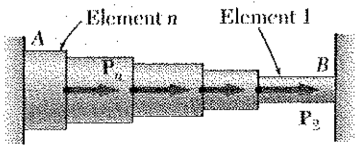
PROBLEM 2.C2



2.C2 Rod *AB* is horizontal with both ends fixed; it consists of *n* elements, each of which is homogeneous and of uniform cross section, and is subjected to the loading shown. The length of element *i* is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (Note that $P_1 = 0$.) (*a*) Write a computer program which can be used to determine the reactions at *A* and *B*, the average normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs. 2.41 and 2.42.

SOLUTION

```
WE CONSIDER THE REACTION AT B REDUNDANT
AND RELEASE THE ROD AT B
COMPUTE $\delta_B$ WITH $R_B = 0$
    FOR EACH ELEMENT, ENTER
        $L_i$, $A_i$, $E_i$
    UPDATE AXIAL LOAD
        $P = P + P_i$
    COMPUTE FOR EACH ELEMENT
        $\sigma_i = P/A_i$
        $\delta_i = PL_i/A_iE_i$
    UPDATE TOTAL DEFORMATION
        $\delta_B = \delta_B + \delta_i$
COMPUTE $\delta_B$ DUE TO UNIT LOAD AT B
        UNIT $\sigma_i = 1/A_i$
        UNIT $\delta_i = L_i/A_iE_i$
    UPDATE TOTAL UNIT DEFORMATION
        UNIT $\delta_B$ = UNIT $\delta_B$ + UNIT $\delta_i$
SUPERPOSITION
    FOR TOTAL DISPLACEMENT AT B = ZERO
        $\delta_B + R_B$ UNIT $\delta_B = 0$
    SOLVING:
        $R_B = -\delta_B/$UNIT $\delta_B$
    THEN:
        $R_A = \Sigma P_i + R_B$
```

CONTINUED

FOR EACH ELEMENT

$$\sigma = \sigma_i + R_B \text{ UNIT } \sigma_i$$

$$\delta = \delta_i + R_B \text{ UNIT } \delta_i$$

PROGRAM OUTPUT

```
Problem  2.41
     RA =  -62.809 kN
     RB =  -37.191 kN
   Element  Stress (MPa)  Deformation (mm)

     1       -52.615          -.05011
     2         3.974           .00378
     3         2.235           .00134
     4        49.982           .04498

Problem  2.42
     RA =  -45.479 kN
     RB =  -54.521 kN
   Element  Stress (MPa)  Deformation (mm)

     1       -77.131          -.03857
     2       -20.542          -.01027
     3       -11.555          -.01321
     4        36.191           .06204
```

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PROBLEM 2.C3

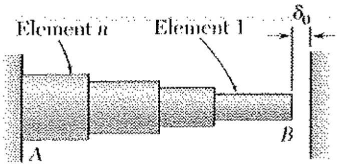

Fig. P2.C3

2.C3 Rod AB consists of n elements, each of which is homogeneous and of uniform cross section. End A is fixed, while initially there is a gap δ_0 between end B and the fixed vertical surface on the right. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and its coefficient of thermal expansion by α_i. After the temperature of the rod has been increased by ΔT, the gap at B is closed and the vertical surfaces exert equal and opposite forces on the rod. (*a*) Write a computer program which can be used to determine the magnitude of the reactions at A and B, the normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs. 2.51, 2.59, and 2.60.

SOLUTION

WE COMPUTE THE DISPLACEMENTS AT B
ASSUMING THERE IS NO SUPPORT AT B:

ENTER L_i, A_i, E_i, α_i

ENTER TEMPERATURE CHANGE T

COMPUTE FOR EACH ELEMENT

$$\delta_i = \alpha_i L_i T$$

UPDATE TOTAL DEFORMATION

$$\delta_B = \delta_B + \delta_i$$

COMPUTE δ_B DUE TO UNIT LOAD AT B

$$\text{UNIT } \delta_i = L_i / A_i E_i$$

UPDATE TOTAL UNIT DEFORMATION

$$\text{UNIT } \delta_B = \text{UNIT } \delta_B + \text{UNIT } \delta_i$$

COMPUTE REACTIONS

FROM SUPERPOSITION

$$R_B = (\delta_B - \delta_0) / \text{UNIT } \delta_B$$

THEN

$$R_A = - R_B$$

FOR EACH ELEMENT

$$\sigma_i = - R_B / A_i$$

$$\delta_i = \alpha_i L_i T + R_B L_i / A_i E_i$$

CONTINUED

PROGRAM OUTPUT

```
Problem  2.51
 R =    125.628 kN

 Element     Stress (MPa)    Deform.(microm)

    1          -44.432           .500
    2          -99.972          -.500

Problem  2.59
 R =     172.835 kN

 Element     Stress (MPa)    Deform.(mm)

    1          -115.223         0.236
    2           -96.019         0.24

Problem  2.60
 R =    232.390 kN

 Element     Stress (MPa)    Deform.(microm)

    1         -116.195        363.220
    2         -290.487        136.780
```

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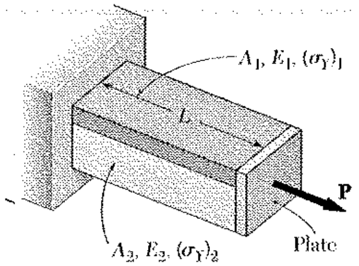
PROBLEM 2.C4



2.C4 Bar AB has a length L and is made of two different materials of given cross-sectional area, modulus of elasticity, and yield strength. The bar is subjected as shown to a load **P** which is gradually increased from zero until the deformation of the bar has reached a maximum value δ_m and then decreased back to zero. (*a*) Write a computer program which, for each of 25 values of δ_m equally spaced over a range extending from 0 to a value equal to 120% of the deformation causing both materials to yield, can be used to determine the maximum value P_m of the load, the maximum normal stress in each material, the permanent deformation δ_p of the bar, and the residual stress in each material. (*b*) Use this program to solve Probs. 2.111, and 2.112.

SOLUTION

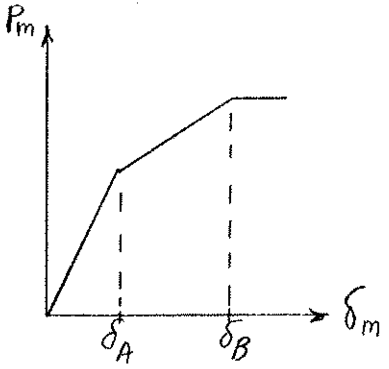
NOTE: THE FOLLOWING ASSUMES $(\sigma_Y)_1 < (\sigma_Y)_2$

DISPLACEMENT INCREMENT

$$\delta_m = 0.05\,(\sigma_Y)_2\, L/E_2$$

DISPLACEMENTS AT YIELDING

$$\delta_A = (\sigma_Y)_1\, L/E_1 \qquad \delta_B = (\sigma_Y)_2\, L/E_2$$



FOR EACH DISPLACEMENT

IF $\delta_m < \delta_A$:

$$\sigma_1 = \delta_m E_1/L$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = (\delta_m/L)(A_1E_1 + A_2E_2)$$

IF $\delta_A < \delta_m < \delta_B$:

$$\sigma_1 = (\sigma_Y)_1$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = A_1\sigma_1 + (\delta_m/L)A_2E_2$$

IF $\delta_m > \delta_B$:

$$\sigma_1 = (\sigma_Y)_1 \qquad \sigma_2 = (\sigma_Y)_2$$
$$P_m = A_1\sigma_1 + A_2\sigma_2$$

PERMANENT DEFORMATIONS, RESIDUAL STRESSES

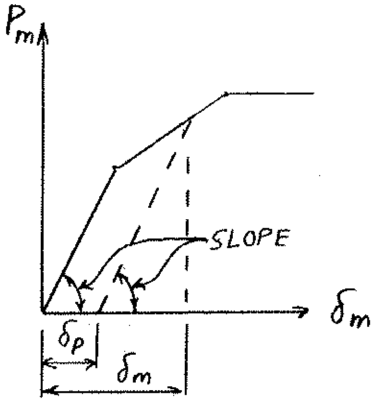
SLOPE OF FIRST (ELASTIC) SEGMENT

$$\text{SLOPE} = (A_1E_1 + A_2E_2)/L$$

$$\delta_p = \delta_m - (P_m/\text{SLOPE})$$

$$(\sigma_1)_{res} = \sigma_1 - (E_1P_m/(L\ \text{SLOPE}))$$

$$(\sigma_2)_{res} = \sigma_2 - (E_2P_m/(L\ \text{SLOPE}))$$



CONTINUED

PROBLEM 2.C4 CONTINUED

PROGRAM OUTPUT

```
         Problems  2.109 and 2.110
 DM       PM       SIGM(1)   SIGM(2)   DP      SIGR(1)   SIGR(2)
 mm       kN       MPa       MPa       mm      MPa       MPa
 0        0        0         0         0       0         0

 0.368    233.571  207.000   207.000   0       0         0

 0.796    436.000  345.000   448.562   0.104   -44.892   58.580   ◄ P2.110

 1.016    495.885  345.000   572.395   0.229   -98.560   128.835  ◄ P2.109

 1.471    556.000  345.000   700.000   0.593   -150.248  198.305
```

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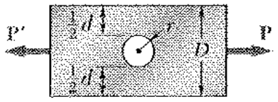
PROBLEM 2.C5



2.C5 The plate has a hole centered across the width. The stress concentration factor for a flat bar under axial loading with a centric hole is:

$$K = 3.00 - 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 - 1.53\left(\frac{2r}{D}\right)^3$$

where r is the radius of the hole and D is the width of the bar. Write a computer program to determine the allowable load **P** for the given values of r, D, the thickness t of the bar, and the allowable stress σ_{all} of the material. Knowing that $r = 6$ mm, D = 75 mm and $\sigma_{\text{all}} = 110$ MPa, determine the allowable load **P** for values of t from 3 mm to 18 mm, using 3 mm increments.

SOLUTION

ENTER

r, D, t, σ_{all}

COMPUTE K

$RD = 2.0\ r/D$

$K = 3.00 - 3.13\ RD + 3.66\ RD^2 - 1.53\ RD^3$

COMPUTE AVERAGE STRESS

$\sigma_{ave} = \sigma_{all}/K$

ALLOWABLE LOAD

$P_{all} = \sigma_{ave}\ (D - 2.0\,r)\,t$

PROGRAM OUTPUT

```
Radius    Allowable Load
 (mm)          (kN)

  3           16.405
  6           16.023
  9           15.368
 12           14.391
 15           13.165
 18           11.721
```

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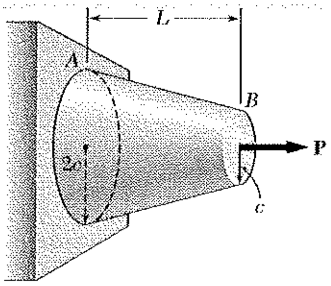
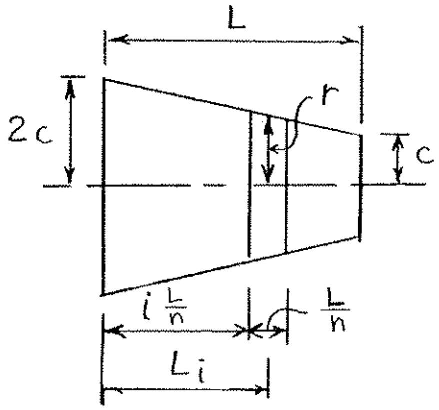
PROBLEM 2.C6



2.C6 A solid truncated cone is subjected to an axial force **P** as shown. The exact elongation is $(PL)/(2\pi c^2 E)$. By replacing the cone by n circular cylinders of equal thickness, write a computer program that can be used to calculate the elongation of the truncated cone. What is the percentage error in the answer obtained from the program using (a) $n = 6$, (b) $n = 12$, (c) $n = 60$.

SOLUTION



```
FOR $i = 1$ TO $n$:
    $L_i = (i + 0.5)(L/n)$
    $r_i = 2c - c(L_i/L)$
    AREA:
        $A = \pi r_i^2$
    DISPLACEMENT:
        $\delta = \delta + P(L/n)/(AE)$
EXACT DISPLACEMENT:
        $\delta_{EXACT} = PL/(2.0\pi c^2 E)$
PERCENTAGE ERROR:
        PERCENT = $100(\delta - \delta_{EXACT})/\delta_{EXACT}$
```

PROGRAM OUTPUT

```
 n       Approximate       Exact      Percent

 6         0.15852        0.15915     -.40083
12         0.15899        0.15915     -.10100
60         0.15915        0.15915     -.00405
```

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